

Exercise 07

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| NAME: | MATRICULATION NUMBER: |
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The exercise is due on Wednesday, June 6, 8 am.

7.1 Text book

Read sections 3.2 to 3.4 and 4.3 in G.H. Findenegg, T. Hellweg „Statistische Thermodynamik“.

7.2 Fermions, bosones, and distinguishable particles

Consider 5 particles in a one-dimensional box (equidistant energy levels).

- (a) Calculate and compare the energy of the ground state for fermions, bosones, and distinguishable particles.
- (b) Suppose that $N_e = 5$. What is the total number of microstates for fermions, bosones, and distinguishable particles?

7.3 Distinguishable particles: two-level system

Consider a particle, which has two spin states, up (+1) and down (-1). If a magnetic field \mathbf{B} is applied, the energy of the two spin states is

$$E(\text{up}) = +\boldsymbol{\mu} \cdot \mathbf{B} ; \quad E(\text{down}) = -\boldsymbol{\mu} \cdot \mathbf{B} \quad (3)$$

where $\boldsymbol{\mu}$ is the magnetic momentum and the energy difference between the two states is given by

$$\Delta E = E(\text{up}) - E(\text{down}) = 2\boldsymbol{\mu} \cdot \mathbf{B} \quad (4)$$

- (a) Write down the partition function Q_1 for a single particle.
- (b) Assume that $E(\text{down}) = 0$, write down the partition function Q_1^0 .
- (c) What is the partition function Q_N^0 for a system of N distinguishable particles?
- (d) Determine the free energy F , the internal energy U , the entropy S and the heat capacity C_V .
- (e) Plot the internal energy and the heat capacity as a function of the temperature.

7.4 Crystals

The table shows the molar heat capacity of crystalin KCl for a range of temperatures:

| T (K) | C_v (J /mol K) |
|---------|------------------|
| 50 | 21.1 |
| 100 | 39.0 |
| 175 | 46.1 |
| 250 | 48.6 |

- (a) Can the Einstein model be applied to this ionic crystal? Hint: Determine the characteristic temperature Θ at $T = 100$ K, and calculate the molar heat capacity at $T = 175$ K.