The most famous single-neuron model: Hodgkin and Huxley

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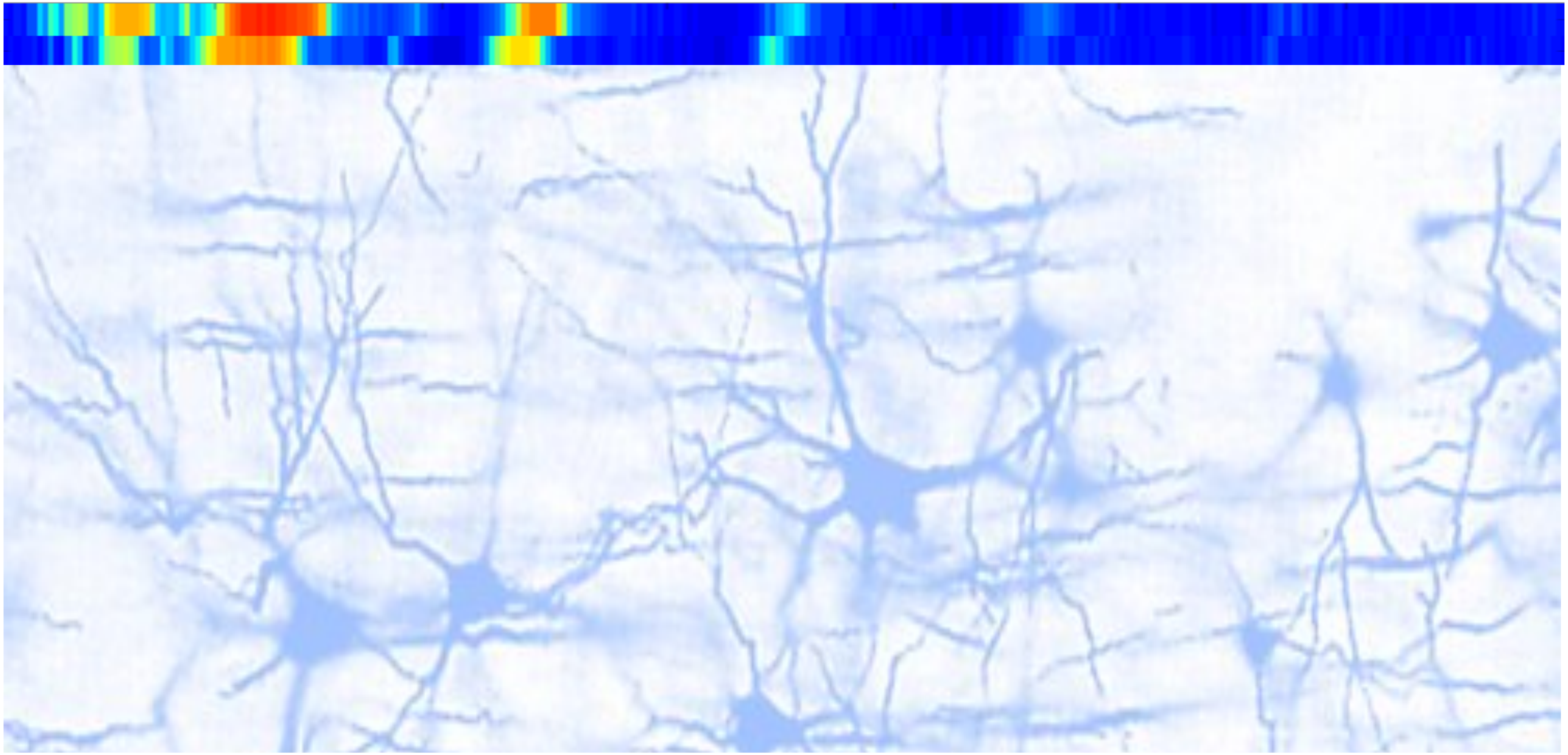

The “Computational Neurophysiology” group

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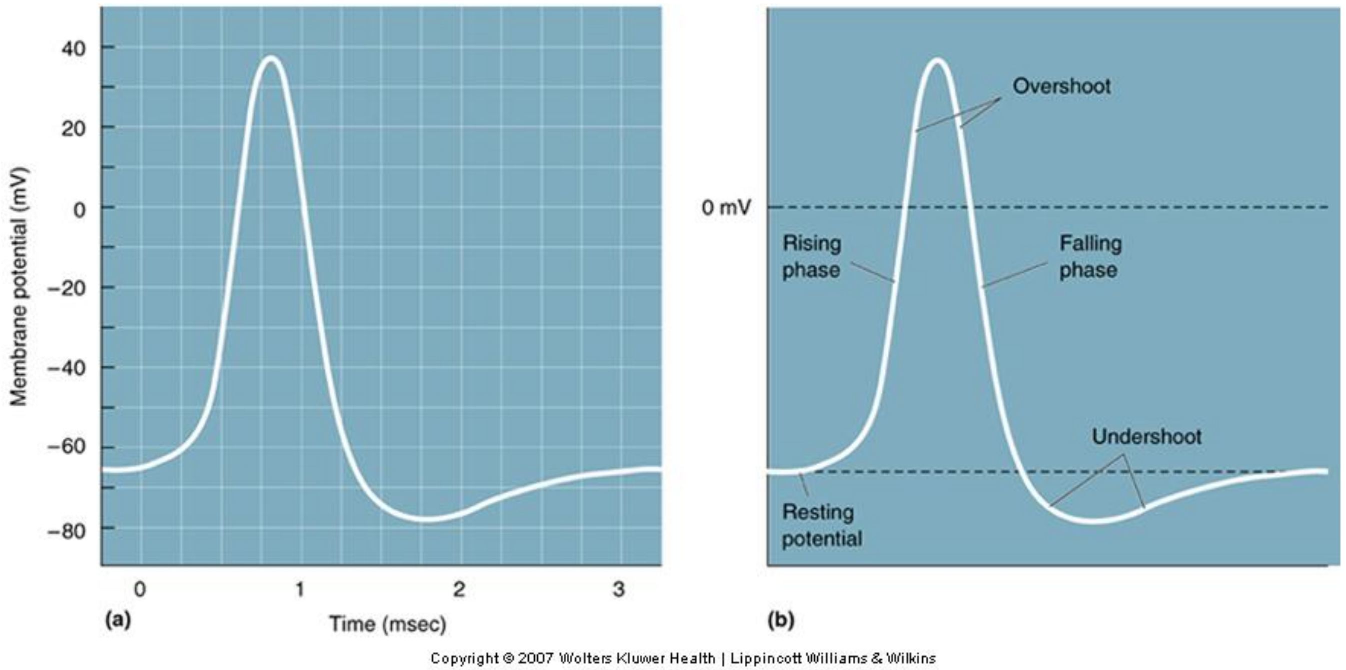
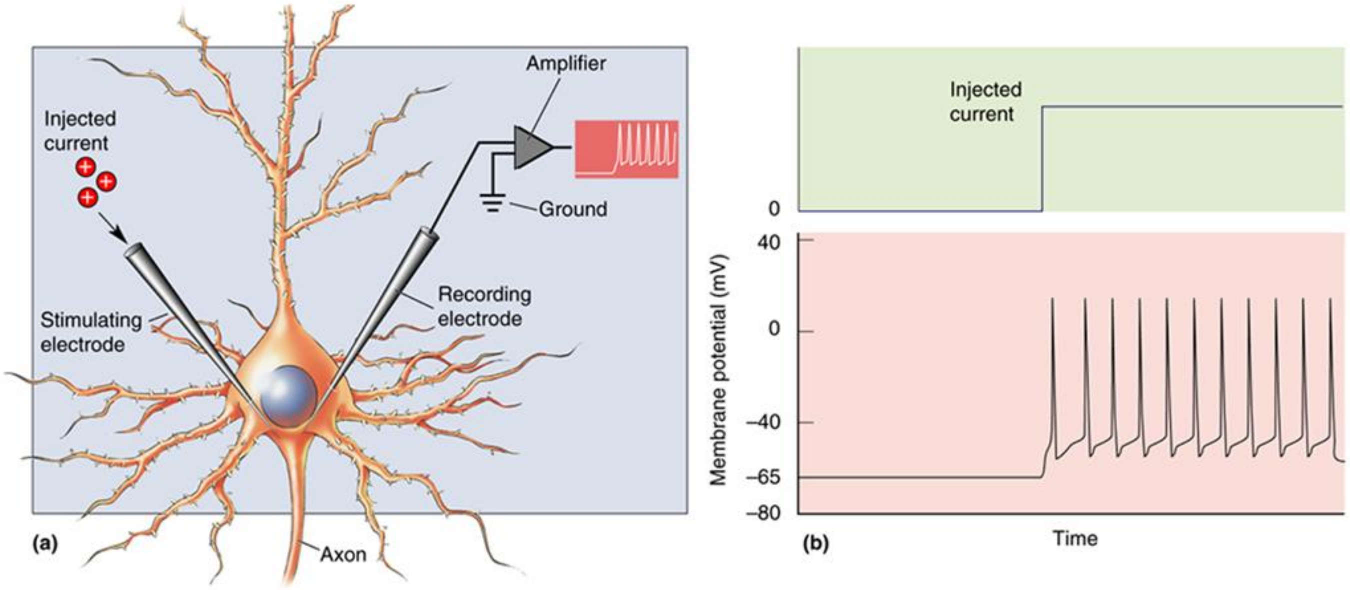
The action potential

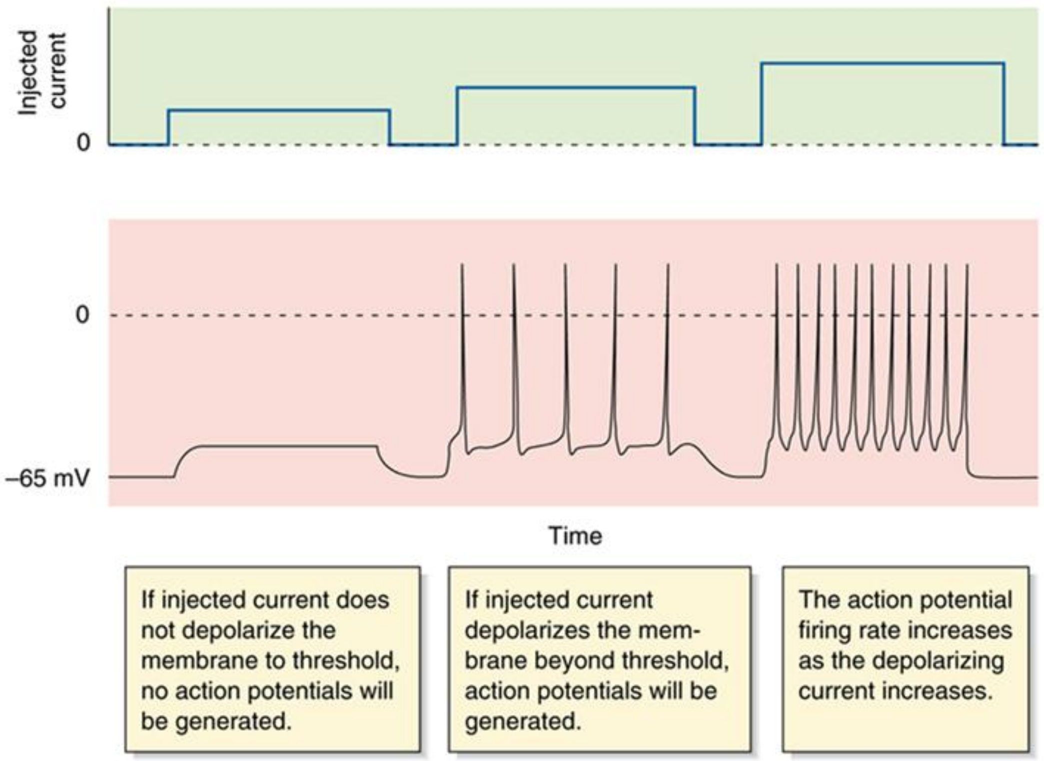


Measuring spikes



Depolarization can elicit regular spiking.

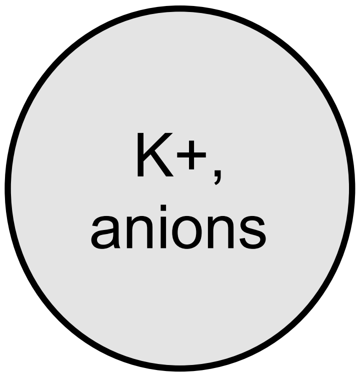
Firing depends on the input

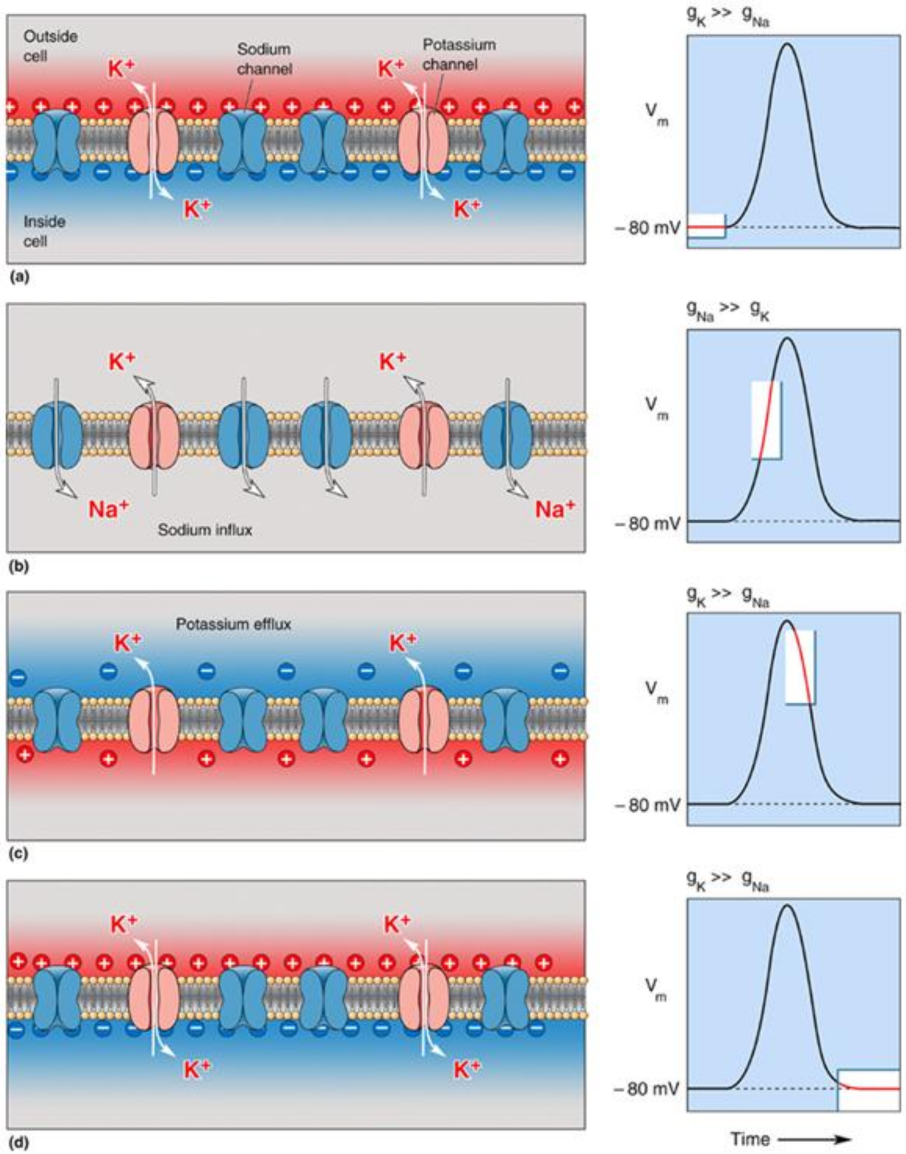


The firing rate depends on the size of the injected current.

Flow of ions during an action potential

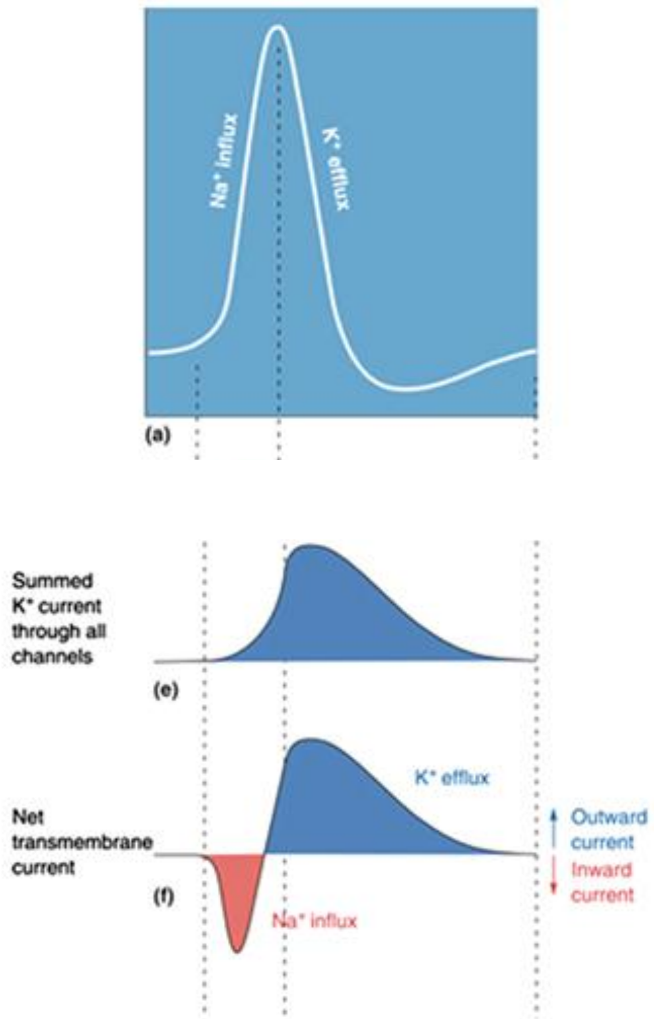
outside: Na^+, Cl^-





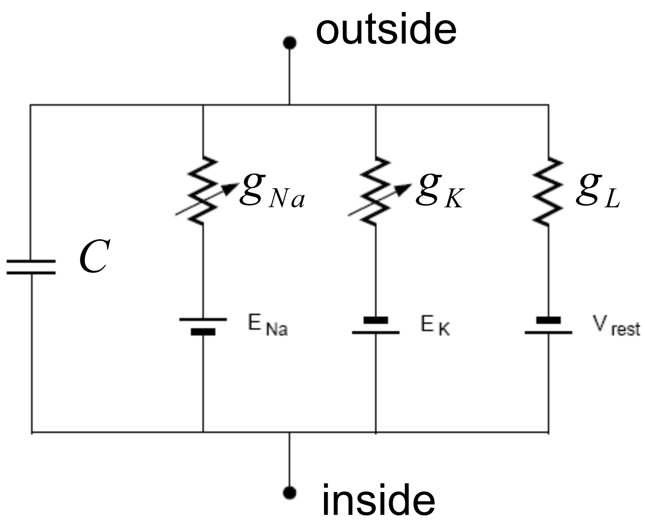
Currents during the AP

During the action potential both sodium and potassium currents flow across the membrane.



The Hodgkin-Huxley Model

The membrane viewed as electrical circuit



Kirchhoff's law:

$$C\frac{dV}{dt} = -I_{Na} - I_K - I_L + I_{input}$$

$$I_{Na} = g_{Na}(V - E_{Na})$$

The reversal potential

potential, where the net flow of ions across the membrane is zero (even if channels were open)

determined by **electric forces** and **diffusion** (inside versus outside charge and concentration)

Nernst equation:

$$E = \frac{V_T}{z} \ln \left(\frac{[\text{outside}]}{[\text{inside}]} \right)$$

The membrane viewed as electrical circuit

$$C\frac{dV}{dt} = -I_{Na} - I_K - I_L + I_{input}$$

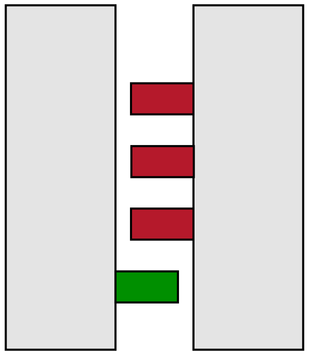
↓

$$C\frac{dV}{dt} = -g_{Na}(V - E_{Na}) - g_K(V - E_K) - g_L(V - E_L) + I_{input}$$

The conductances are not constant, but functions of the voltage V.

Gating particles of sodium and potassium channels

sodium channel:

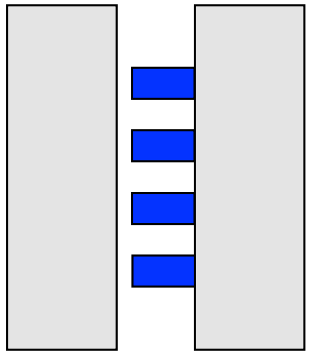


3 x m gating particles

1 x h gating particle

$$\Rightarrow \quad g_{Na} = \overline{g_{Na}} m^3 h$$

potassium channels:



4 x n gating particles

$$g_K = \overline{g_K} n^4$$

The Hodgkin-Huxley voltage equation

$$C\frac{dV}{dt} = -g_{Na}(V - E_{Na}) - g_K(V - E_K) - g_L(V - E_L) + I_{input}$$

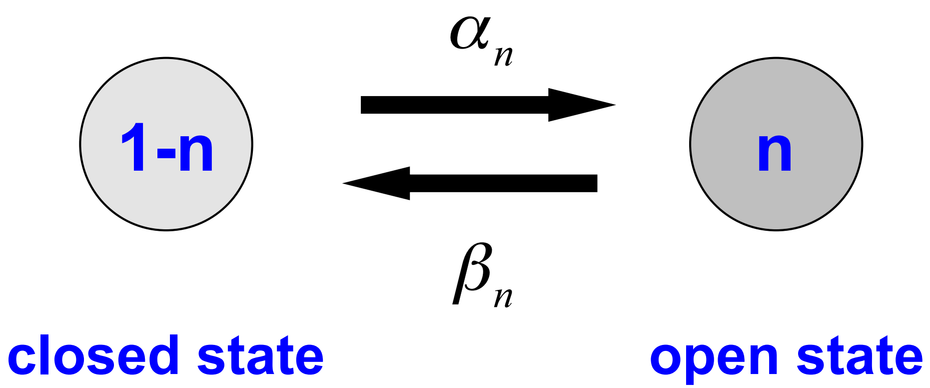
↓

$$C\frac{dV}{dt} = -\overline{g_{Na}}\ m^3 h\ (V - E_{Na}) - \overline{g_K} n^4 (V - E_K) - g_L(V - E_L) + I_{input}$$

But what are the dynamics of m, h, and n?

Linear kinetics for transitions between states

First order kinetics:



Resulting differential equation for dynamics:

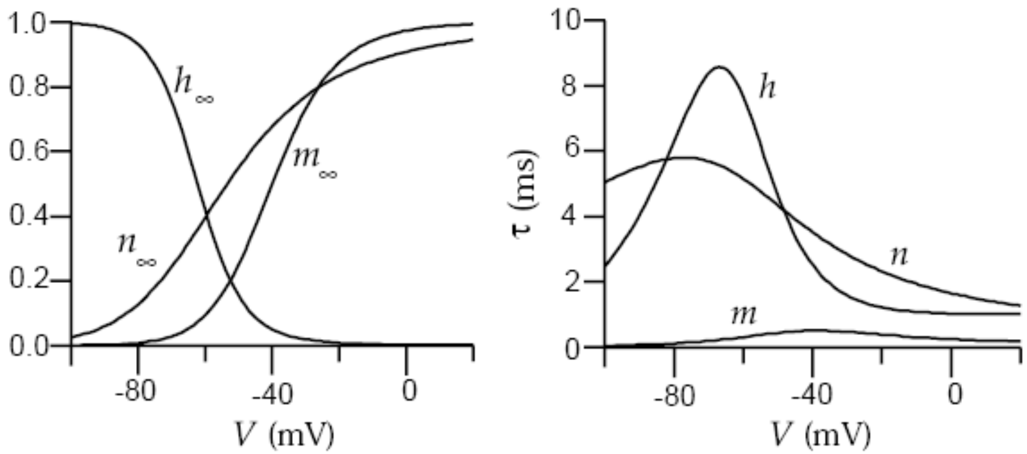
$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

Dynamical properties of the gating variables

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

reformulation:

$$\tau_n(V)\frac{dn}{dt} = n_\infty(V) - n$$



steady state value:

$$n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)}.$$

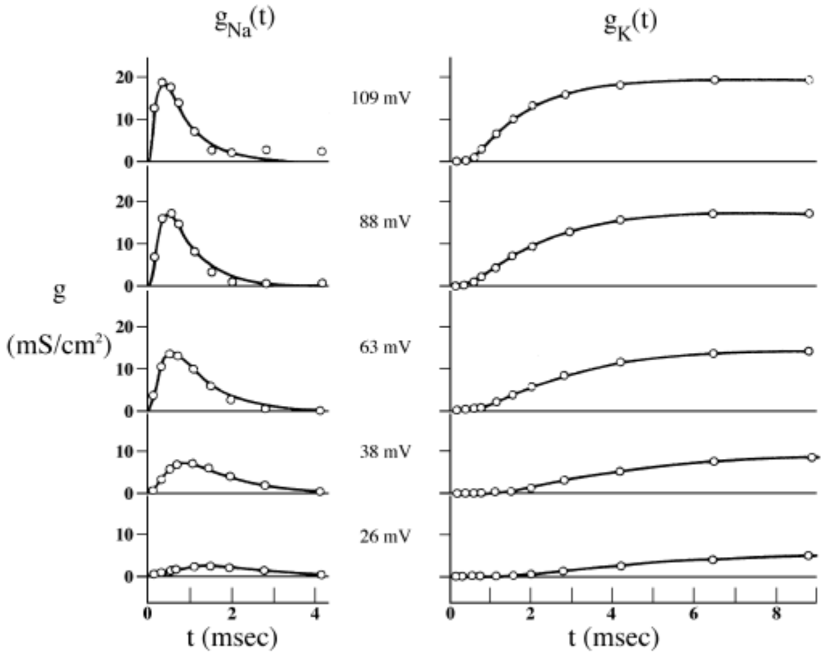
time constant:

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

Measuring rate constants

$$\begin{aligned}\dot{n} &= \alpha_n(V)(1-n) - \beta_n(V)n \\ \dot{m} &= \alpha_m(V)(1-m) - \beta_m(V)m \\ \dot{h} &= \alpha_h(V)(1-h) - \beta_h(V)h\ ,\end{aligned}$$

rate constants were fitted by Hodgkin & Huxley:



$$\begin{aligned}\alpha_n(V) &= 0.01\frac{10-V}{\exp(\frac{10-V}{10})-1}\ , \\ \beta_n(V) &= 0.125\exp\left(\frac{-V}{80}\right)\ , \\ \alpha_m(V) &= 0.1\frac{25-V}{\exp(\frac{25-V}{10})-1}\ , \\ \beta_m(V) &= 4\exp\left(\frac{-V}{18}\right)\ , \\ \alpha_h(V) &= 0.07\exp\left(\frac{-V}{20}\right)\ , \\ \beta_h(V) &= \frac{1}{\exp(\frac{30-V}{10})+1}\ .\end{aligned}$$

The full Hodgkin-Huxley model

system of four differential equations,
coupled via the membrane potential V:

$$
\begin{aligned}
C\dot{V} &= I - \overbrace{\bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}})}^{I_{\mathrm{K}}} - \overbrace{\bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}})}^{I_{\mathrm{Na}}} - \overbrace{g_{\mathrm{L}} (V - E_{\mathrm{L}})}^{I_{\mathrm{L}}} \\
\dot{n} &= \alpha_n(V)(1-n) - \beta_n(V)n \\
\dot{m} &= \alpha_m(V)(1-m) - \beta_m(V)m \\
\dot{h} &= \alpha_h(V)(1-h) - \beta_h(V)h \ ,
\end{aligned}
$$

Conductance-based models

Hodgkin-Huxley model

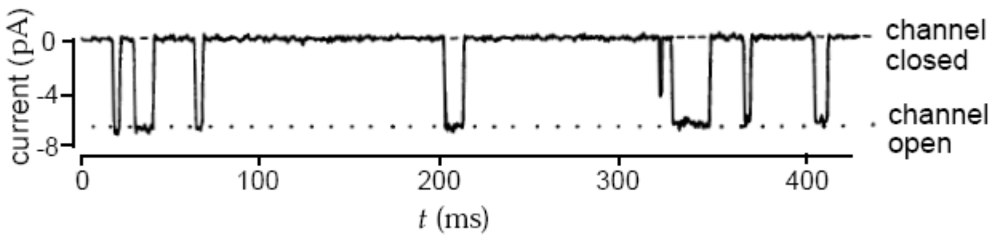
numerical solution of the full deterministic model (tutorial)

further simplification (lower-dimensional models)

stochastic version

Ion channel stochasticity

one channel – two states only:



the more channels, the less noise in the population signal: