Freie Universität Berlin

Theoretical Neuroscience and Neuroinformatics
Institute of Biology

bccn berlin

Bernstein Center for
Computational Neuroscience Berlin

The Myth of Poissonian Spiking

Interval and Count Statistics in Biological Neurons

Martin Paul Nawrot

Workshop on Statistical Aspects of Neural Coding

Kyoto University, Nov 01, 2012

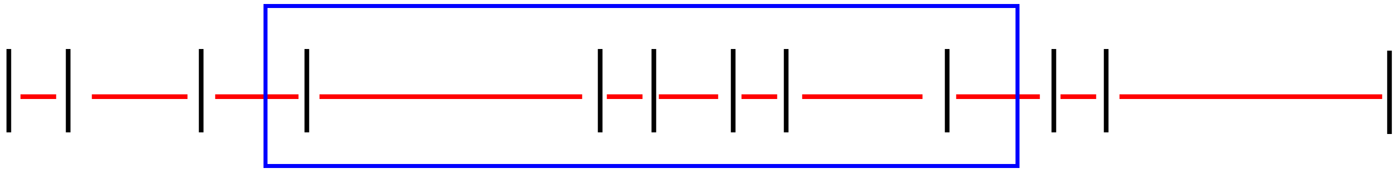
- Introduction
- 1 Measures of 2nd order interval and count statistics
- 2 The Poisson model is a bad model for neural spiking
- 3 Neural spike trains violate assumption of a renewal process
- 4 Non-renewal point process models and biophysics
- Take-home summary

„The variability of neuronal responses is proportional to the mean in many brain areas, which ***suggests that neural responses might follow a Poisson distribution****."*

Averbeck (2009) Neuron 62

2 basic random variables

- inter-event intervals *X* (continuous random variable)
- number of spikes *N* (discrete random variable) in interval of length T



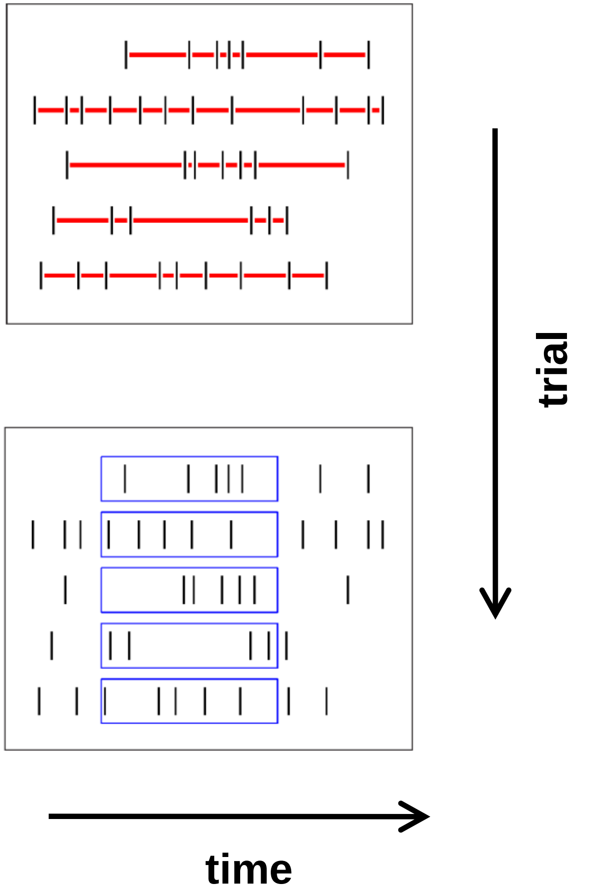
Any point process definition uniquely determines its interval and count stochastic, and both random variables are related.

Coefficient of variation (interval variability)

$$CV^2 = \frac{Var(ISI)}{mean^2(ISI)}$$

Fano factor (count variability)

$$FF = \frac{Var(count)}{mean(count)}$$



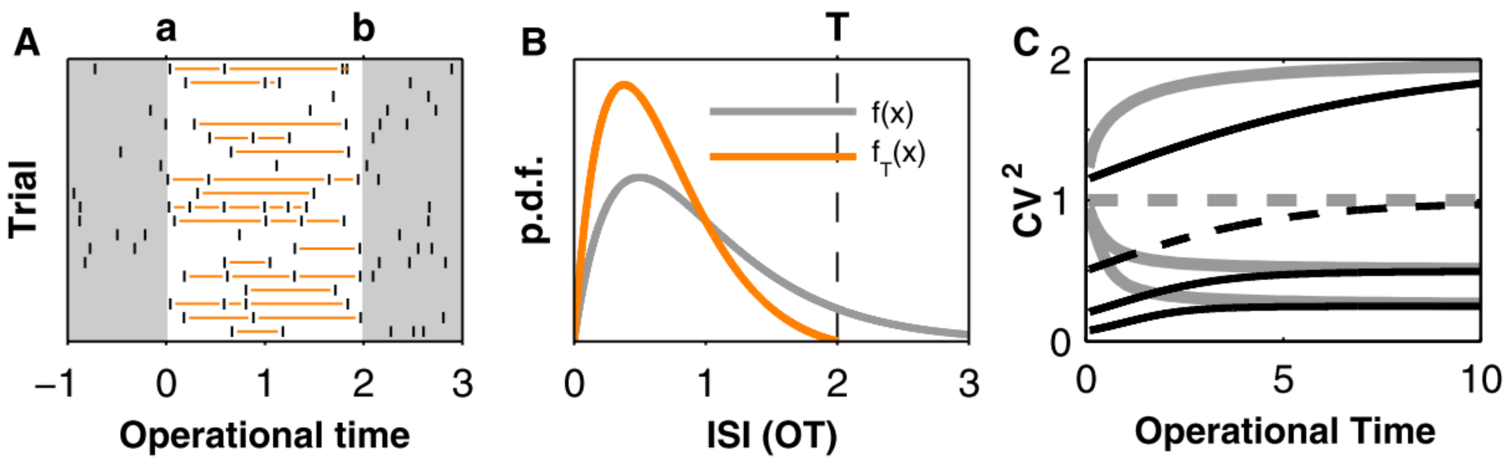
Bias of estimators for finite T

Coefficient of variation
(of inter-spike intervals *X*)

$$CV^2 = \frac{Var(ISI)}{mean^2(ISI)}$$

Fano factor
(of spike count *N*)

$$FF = \frac{Var(count)}{mean(count)}$$



$$\hat{f}(x) = \begin{cases} \eta^{-1}(T-x)\,f(x) & \text{for } x \in [0, T], \\ 0 & \text{otherwise,} \end{cases}$$

$$\eta = \int_0^T (T-s)\,f(s)\,ds$$

Nawrot (2010) In: Grün, Rotter (eds.), Springer Series Comp Neurosci 7

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General definition of a Point Process

One possibility to define a point process is the **complete intensity function**.

Consider a point process as defined on the complete time axis $(-\infty,+\infty)$. Let H_t denote the **history of the process**, i.e. a specification of the position of all points in $(-\infty, t]$. Then a general description of this process maybe formulated in terms of the probabilities of observing a single event at an arbitrary time t

$$P(N(t, t + \delta t) = 1 | H_t\}$$

The dependence on the process history can be arbitrarily complex

The myth of Poissonian spiking

Poisson

- exponential interval distribution
- Poisson count distribution
- events are uniformly distributed in time
- special case of gamma process

$$FF = CV^2 = 1$$

Poisson process has no history dependence

The **Poisson process** of intensity λ is defined by the requirements that for all t and for $\delta \to 0+$

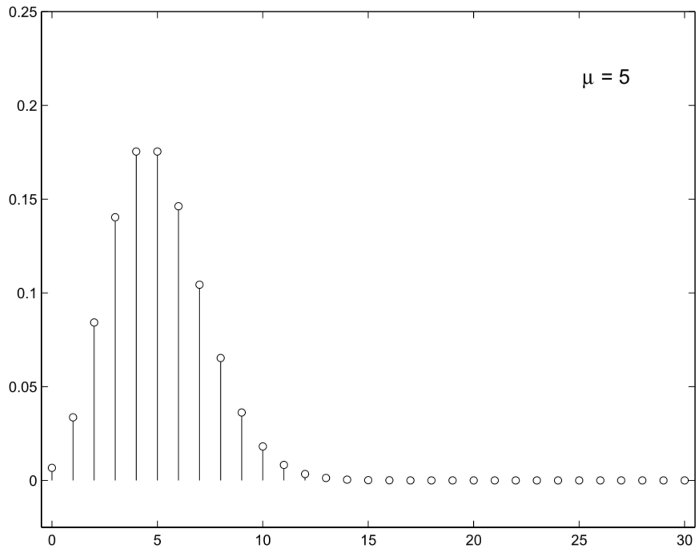
$$P\{N(t, t+\delta t) = 1 | H_t\} = \lambda\delta + o(\delta)$$

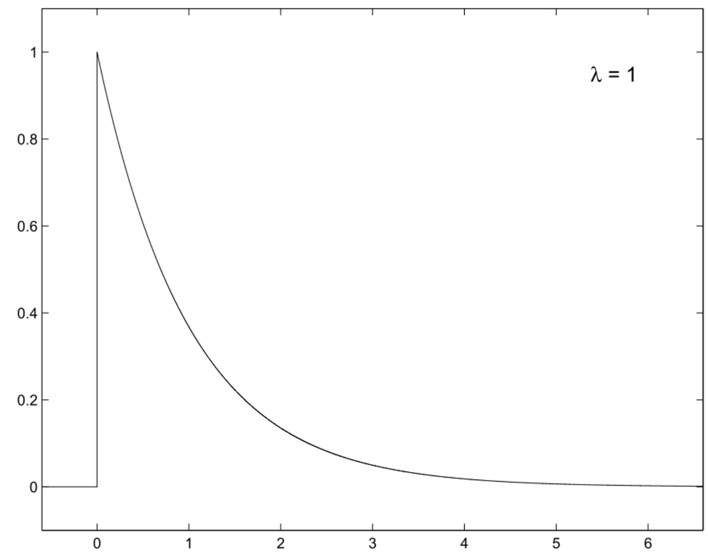
The Poisson process is the only process for which

- all events are completely independent

- the hazard rate is flat (maximizes entropy)

Poisson Count and Interval Distribution





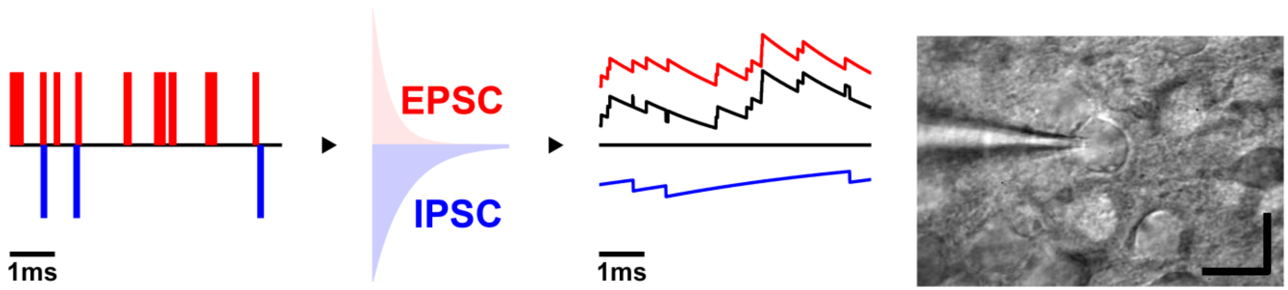
$$P\{N(A)=k\} = \frac{(\lambda t)^k}{k!}\, e^{-\lambda t}$$

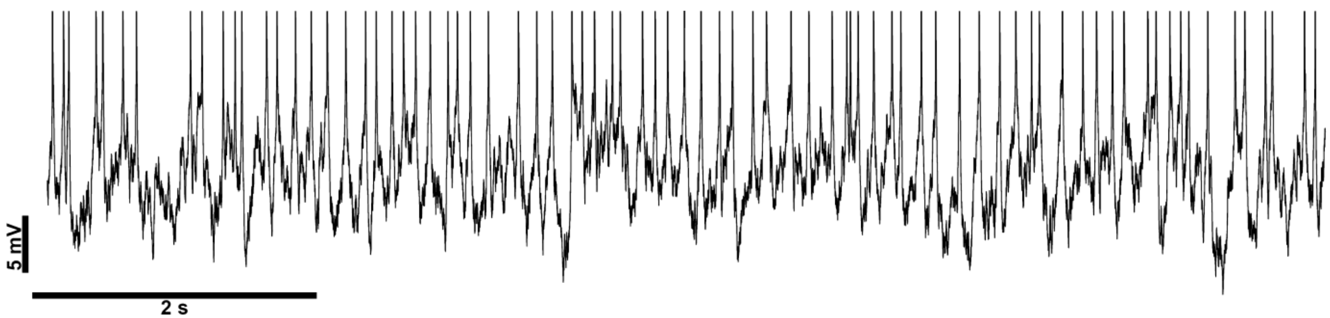
$$\Pr\{X_1 > t\} = P\{N(t)=0\} = \frac{\lambda t^0}{0!}\, e^{-\lambda t} = e^{-\lambda t}$$

FF = 1

CV^2 = 1

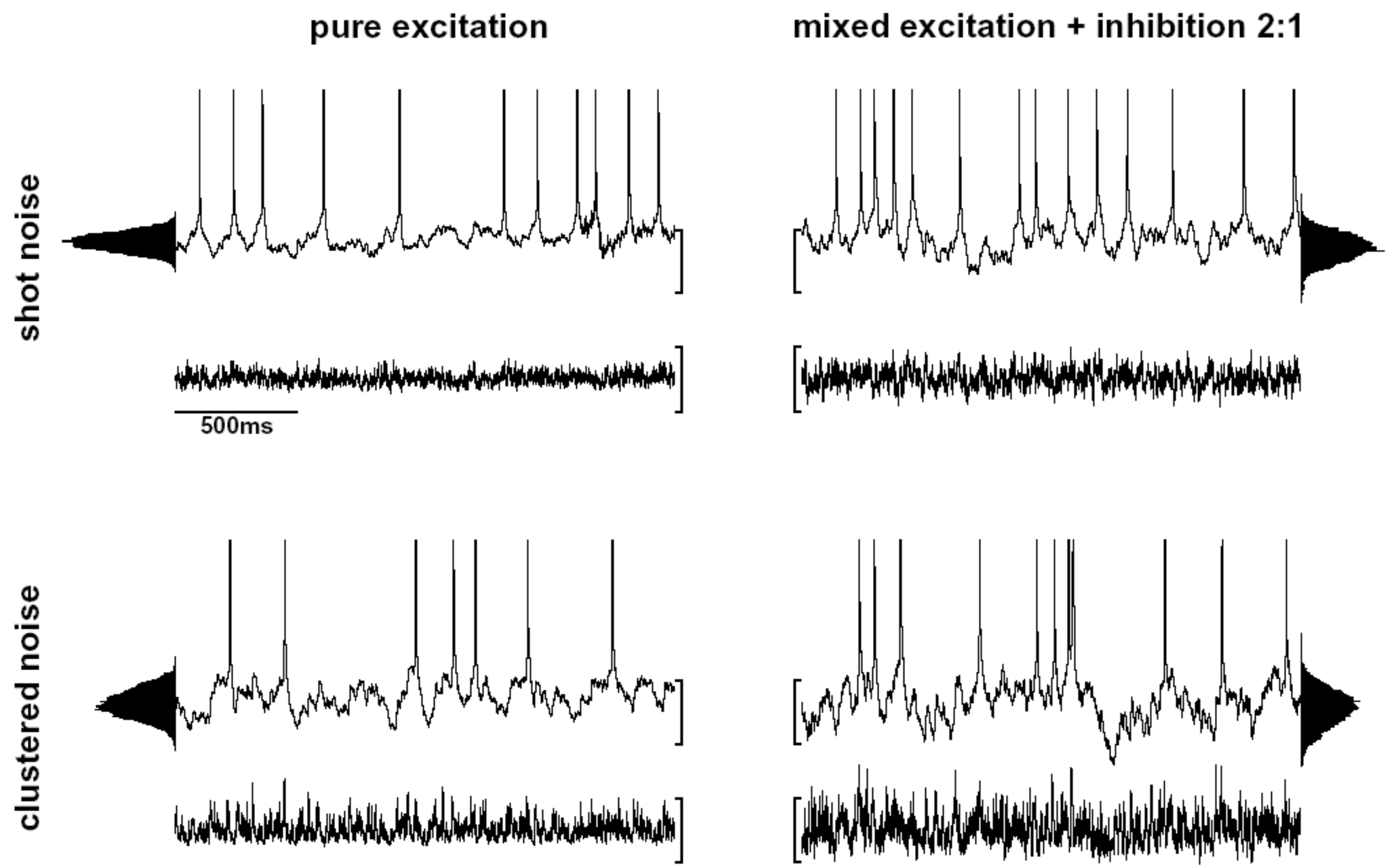
In vitro experiments: Controlled stationary input





Nawrot et al. (2008) J Neurosci Meth

Controlled starionary noise input *in vitro*



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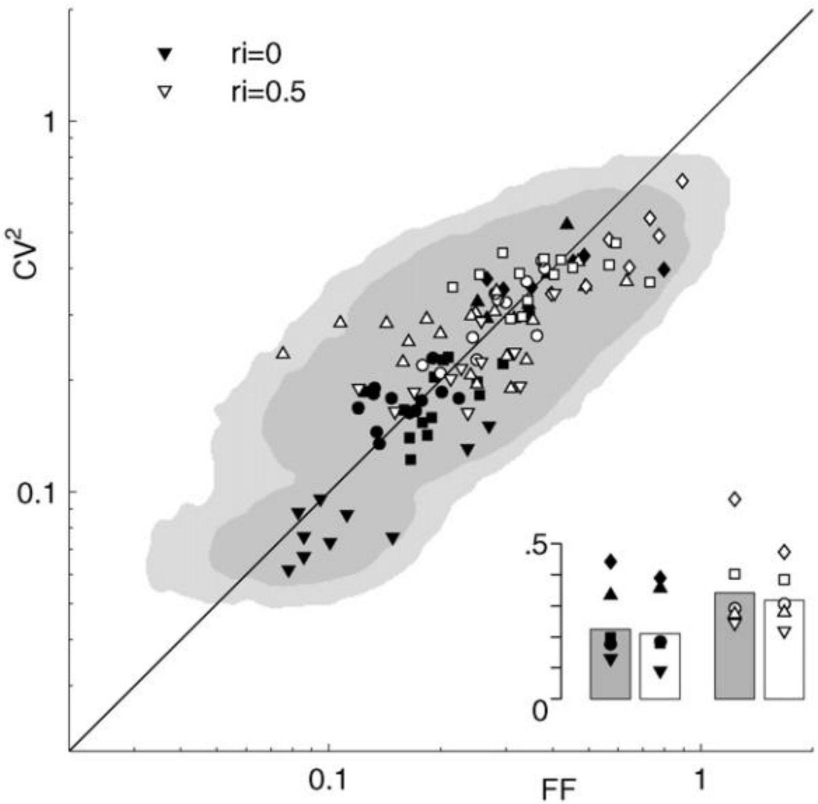
► Cortical neuron *in vitro* show sub-Poissonian interval and count variability



$$FF \approx CV^2 \leq 0.4$$

CV *- Holt et al. (1996) J Neurophysiol 75; Nowak et al. (1997) Cereb Cortex 7; Stevens & Zador (1998) Nat Neurosci 1; Harsch & Robinson (2000) J Neurosci 20; Badoual et al. (2005) Neurocomp 65; Arsiero et al. (2007) J Neurosci 27; Shinomoto*

FF *- Stevens & Zador (1998) Nat Neurosci 1; Harsch & Robinson (2000) J Neurosci 20*

Nawrot et al. (2008) J Neurosci Meth

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Renewal model

constant intensity λ

Poisson

- exponential interval distribution
- Poisson count distribution
- events are uniformly distributed in time
- special case of gamma process

$$FF = CV^2 = 1$$

Renewal

- iid interval distribution
- $FF = CV^2$

$$FF = CV^2$$

increasing importance of process history

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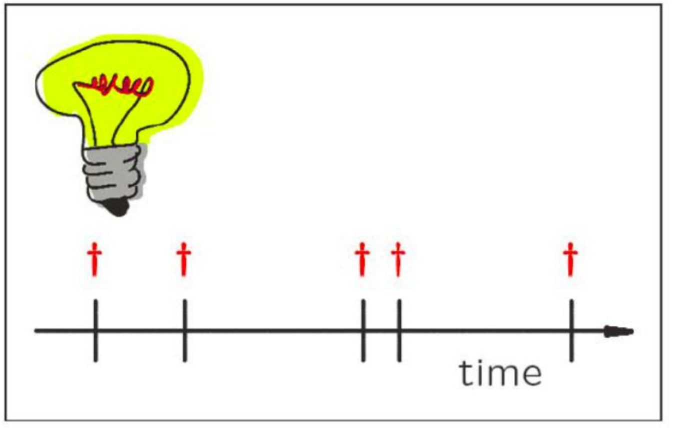
Definition

inter-event intervals are **independent** and **identically distributed** (iid)

Thus

• individual intervals are serially independent
• process history is relevant only up to the previous event
• the intervals between successive points are mutually independent
• the Poisson process is a renewal process



† = replacement from a homogeneous population

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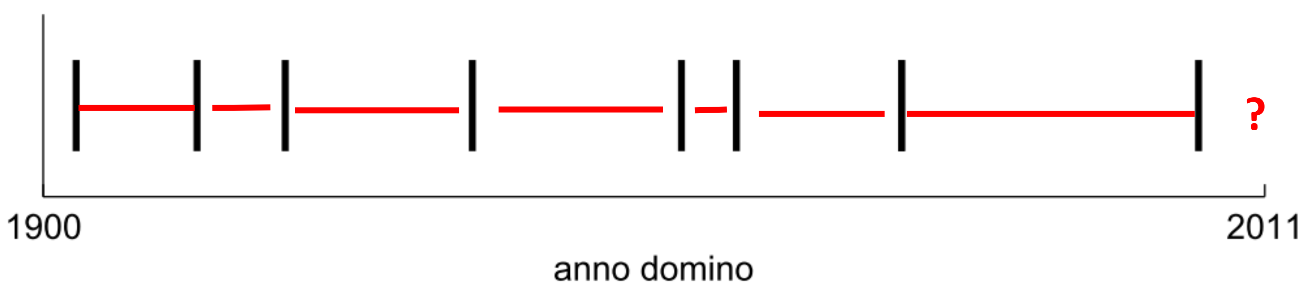
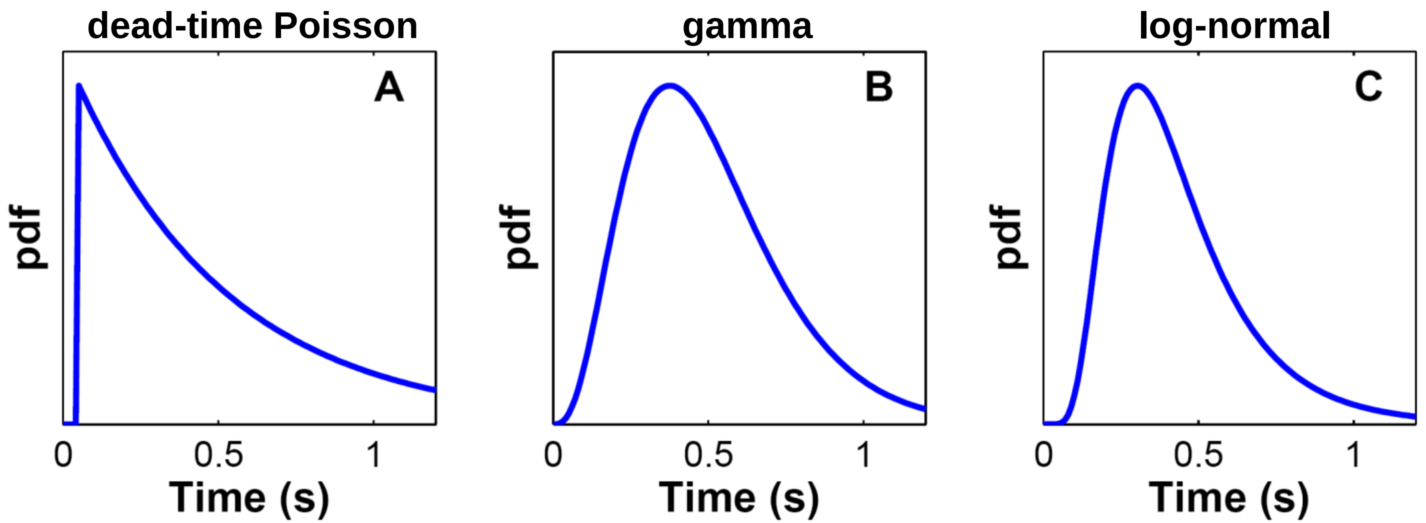
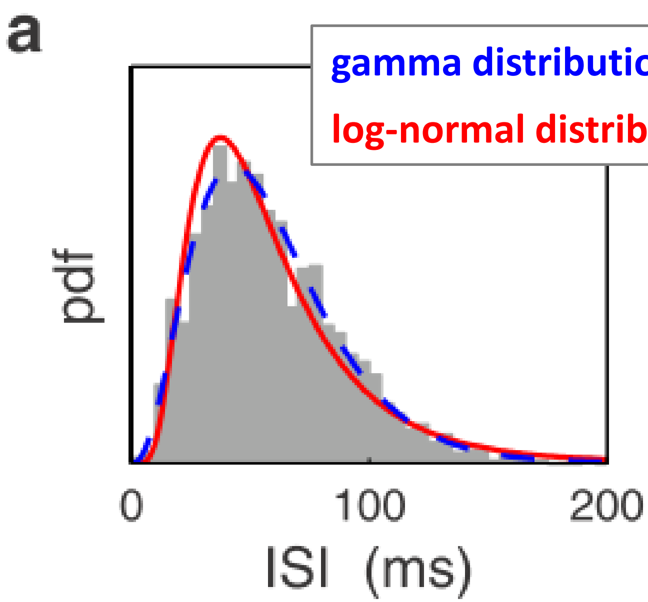
$N_{>1500} = 49$

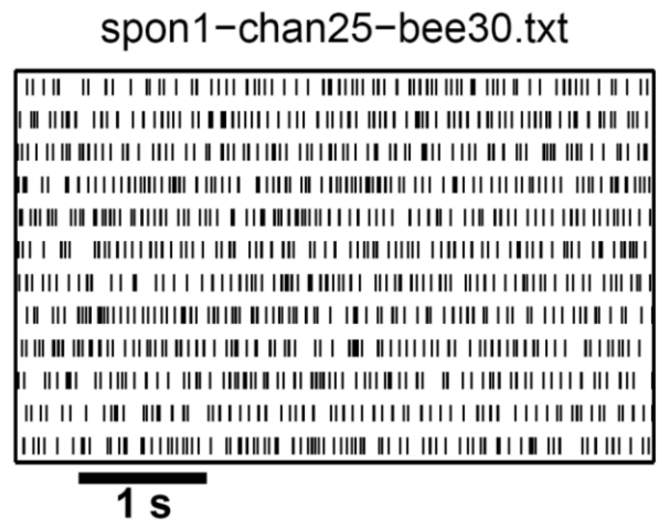
$CV\ (\Delta_{POPE}) = 0.79$



Prominent **interval** distributions used for renewal models of neural spiking







Extracellular recordings of spontaneous activity from mushroom body extrinsic neurons in the Honeybee

Farkhooi, Strube-Bloss, Nawrot (2009) Phys Rev E

Testing the renewal model *in vivo*

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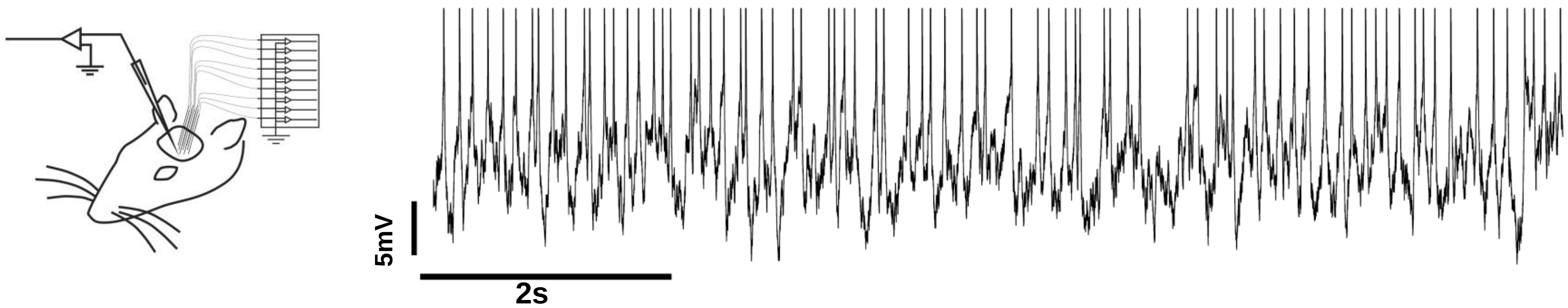
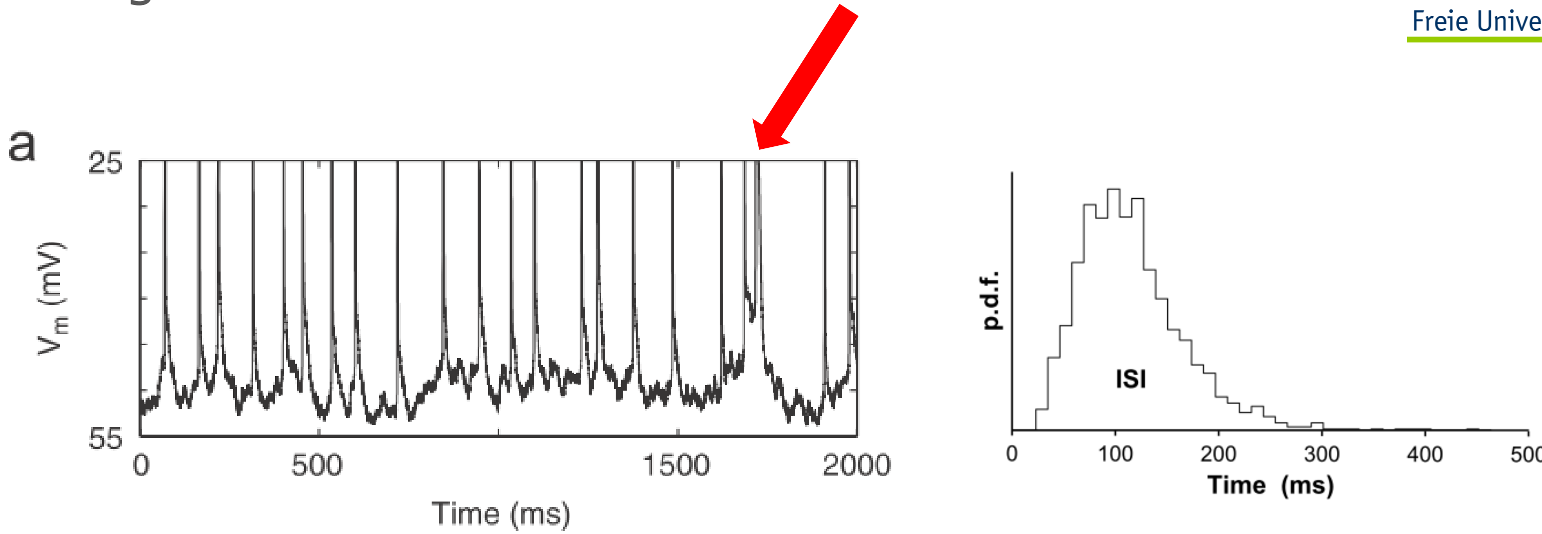
Intracellular recordings form cortical neurons in the somatosensory cortex of the anaesthetized rat

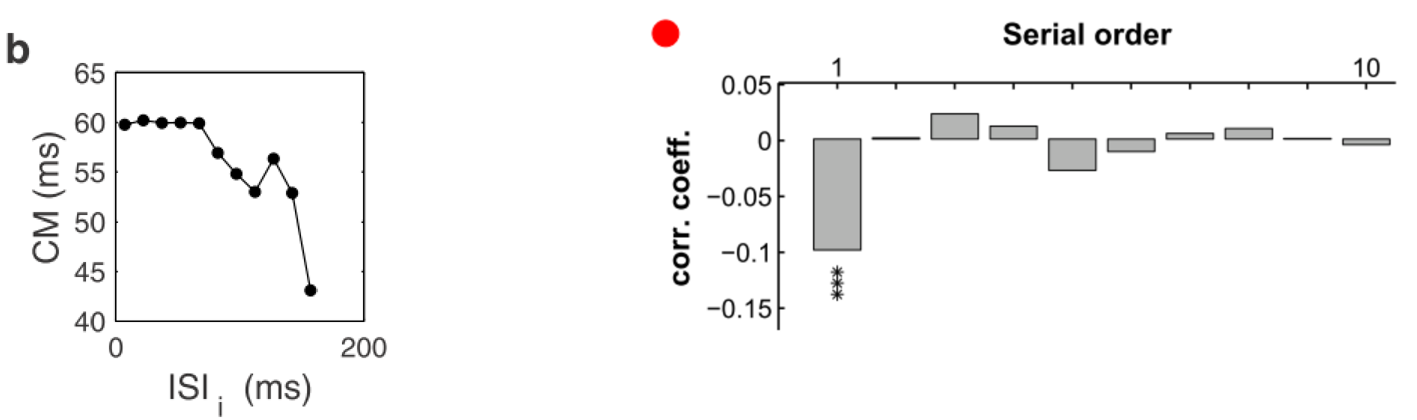


Experiments by ***Clemens Boucsein*** *&* ***Dymphie Suchanek***

University of Freiburg, Germany

Testing the renewal model *in vivo*





Nawrot et al. (2007) Neurocomputing

Testing the renewal model *in vivo*

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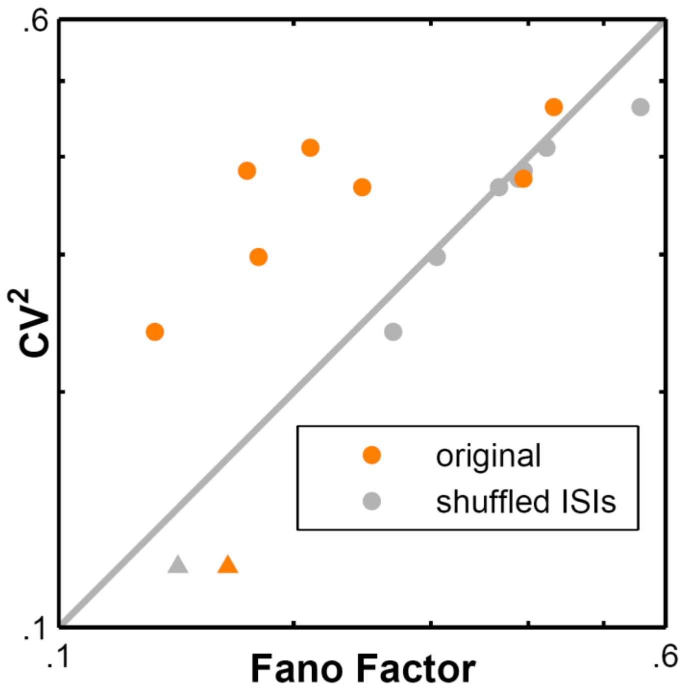
► negative serial correlations reduce the Fano factor (FF < CV^2)



$FF < CV^2 \leq 0.5$

Nawrot et al. (2007) Neurocomputing

$$\lim_{T \to \infty} FF = CV^2 \left(1 + 2 \sum_{i=1}^{\infty} SRC_i \right)$$

Cox & Lewis (1966) Methuen

Experimental evidence for serial ISI correlation

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Reference	Model System & Neuron Type	SC
Ratnam and Nelson (2000)	Weak electric fish, isolated P-type Receptors afferent	-0.52
Chacron et al. (2000)	Weak electric fish, isolated P-type Receptors afferent	-0.35
Neiman and Russell (2004)	Paddle fish, sensory Ganglion	$\sim$ -0.4
Floyd et al. (1982)	Cat splanchnic and hypogastric nerves *in vivo*	-0.3
Levine (1996)	Goldfish retina, Ganglion cells *in vivo*	-0.13
Rodieck (1967)	Cat Retina, Ganglion cells *in vivo*	-0.06
Kuffler et al. (1957)	Cat Retina, Ganglion cells *in vivo*	-0.17
Tsuchitani and Johnson (1985)	Cat Lateral Superior Olive *in vivo*	-0.2
Nawrot et al. (2007)	Rat Somatosensory Cortex (S1) *in vivo*, regular spiking cells	-0.21
Nawrot et al. (2007)	Rat Somatosensory Cortex (S1) *in vitro*, pyramidal cells	-0.07
Engel et al. (2008)	Rat medial entorhinal cortex *in vitro* Layer II stellate and Layer III pyramidal neurons	[-0.1,-0.4]
Farkhooi et al. (2008)	Honeybee central brain *in vivo* Mushroom body extrinsic neurons	-0.15

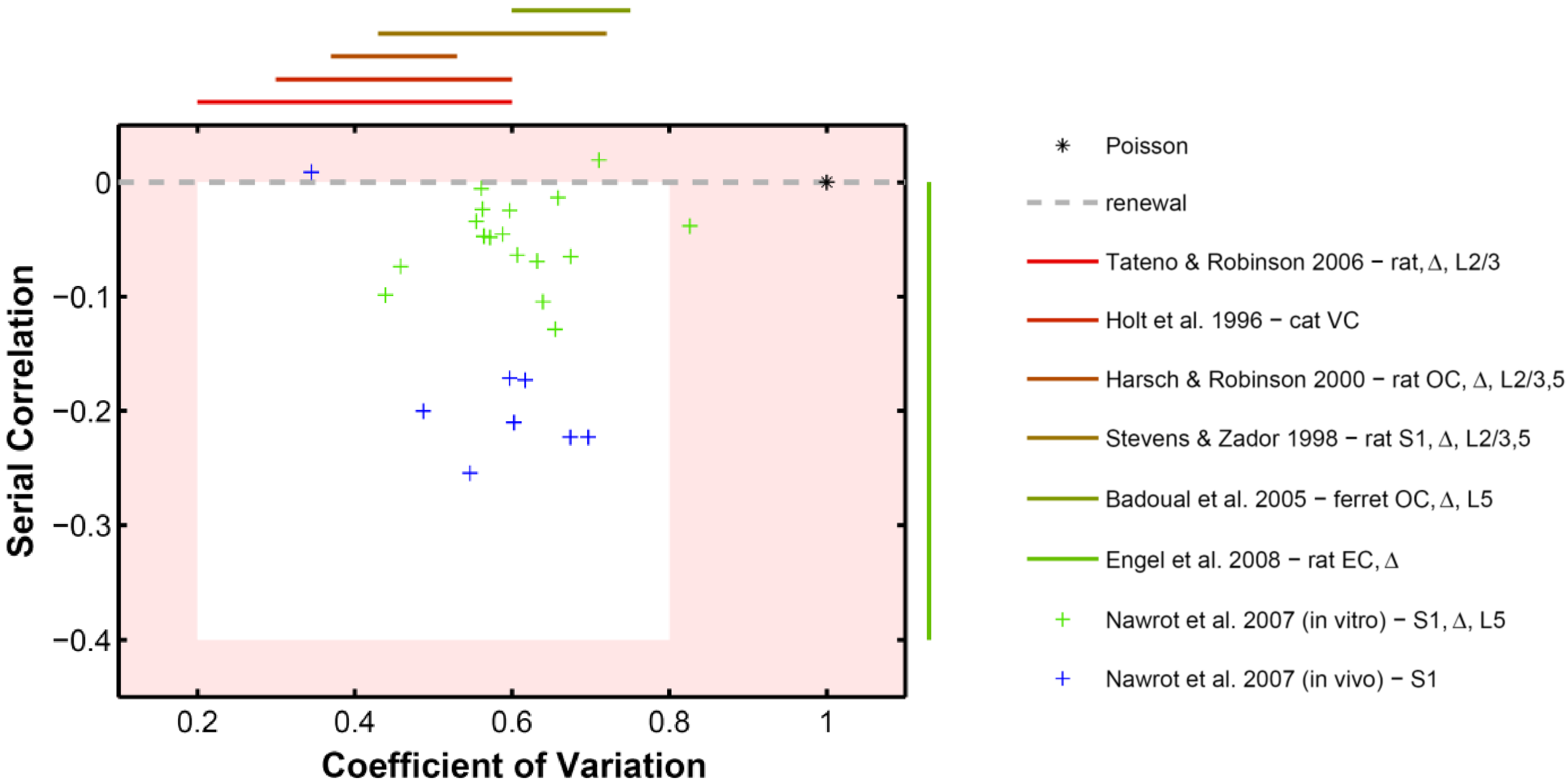
PERIPHERAL

CENTRAL

Table 1: Negative 1st order serial interval correlation in different preparations and cell types.

Farkhooi, Strube-Bloss, Nawrot (2009) Phys Rev E

Estimated parameters in the literature (neocortex)



Nawrot & Grün (in prep)

Non-renewal models

constant intensity λ

Poisson

- exponential interval distribution
- Poisson count distribution
- events are uniformly distributed in time
- special case of gamma process

$$FF = CV^2 = 1$$

Renewal

- iid interval distribution
- $FF = CV^2$

$$FF = CV^2$$

stationary non-Renewal

- constant intensity parameter
- non-trivial history dependence
- serial interval correlations

$$\lim_{T\to\infty} FF = CV^2\left(1 + 2\sum_{i=1}^{\infty} SRC_i\right)$$

increasing importance of process history

Non-renewal autoregressive point process

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Autoregressive model approach

The autoregressive process in the general linear for up to lag *p* reads

$$X_s = \beta_1 X_{s-1} + \beta_2 X_{s-2} + \ldots + \beta_p X_{s-p} + \varepsilon_s$$

where

ε_s i.i.d. with specific mean and finite variance.

β_i correlation coefficient for lag i and $|\beta| < 1$

We propose the following process to model inter-event intervals

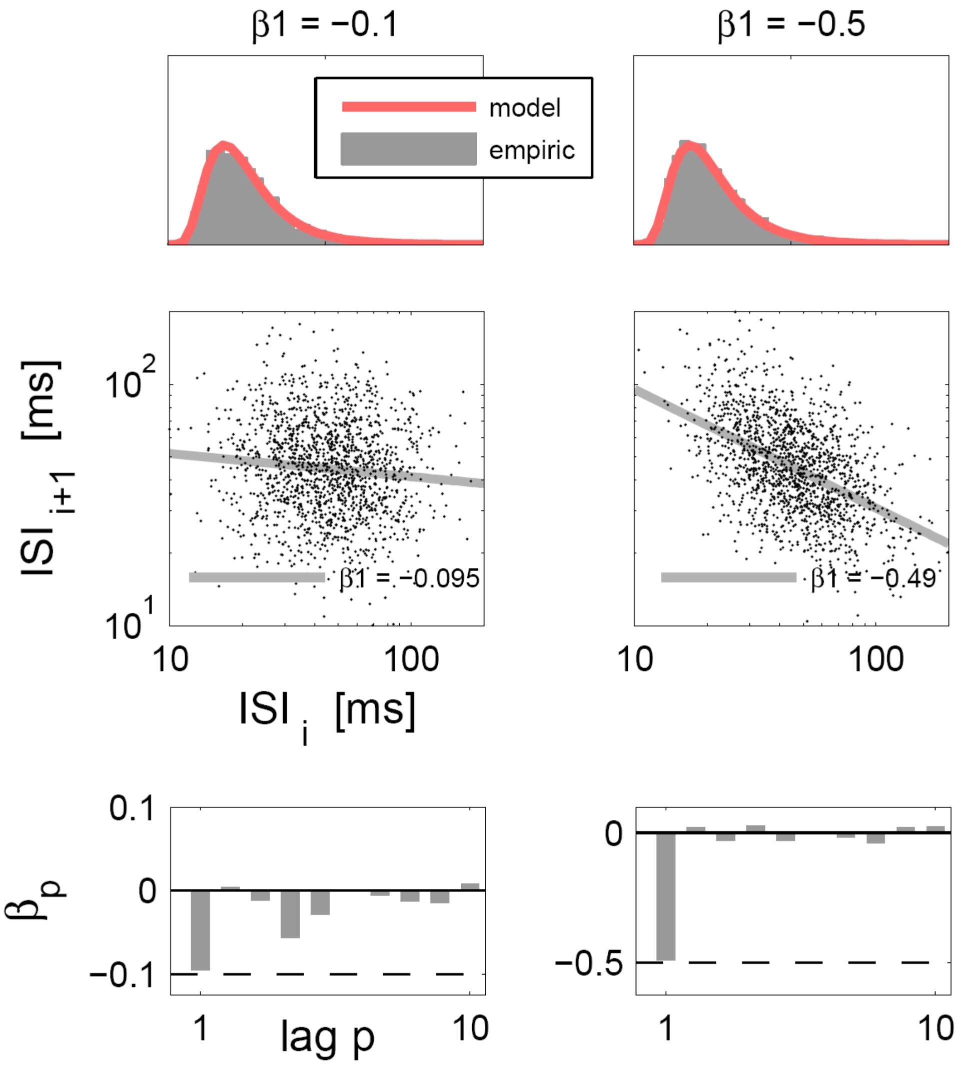
$$\Delta_s = \exp(X_s) = \exp(\beta X_{s-1} + \varepsilon_s) \qquad (|\ \beta\ | < 1)$$

When we choose ε_s *normal distributed* with mean μ and variance σ^2 then Δ_s *is log-normal* distributed.

Farkhooi, Strube-Bloss & Nawrot (2009) Phys Rev E

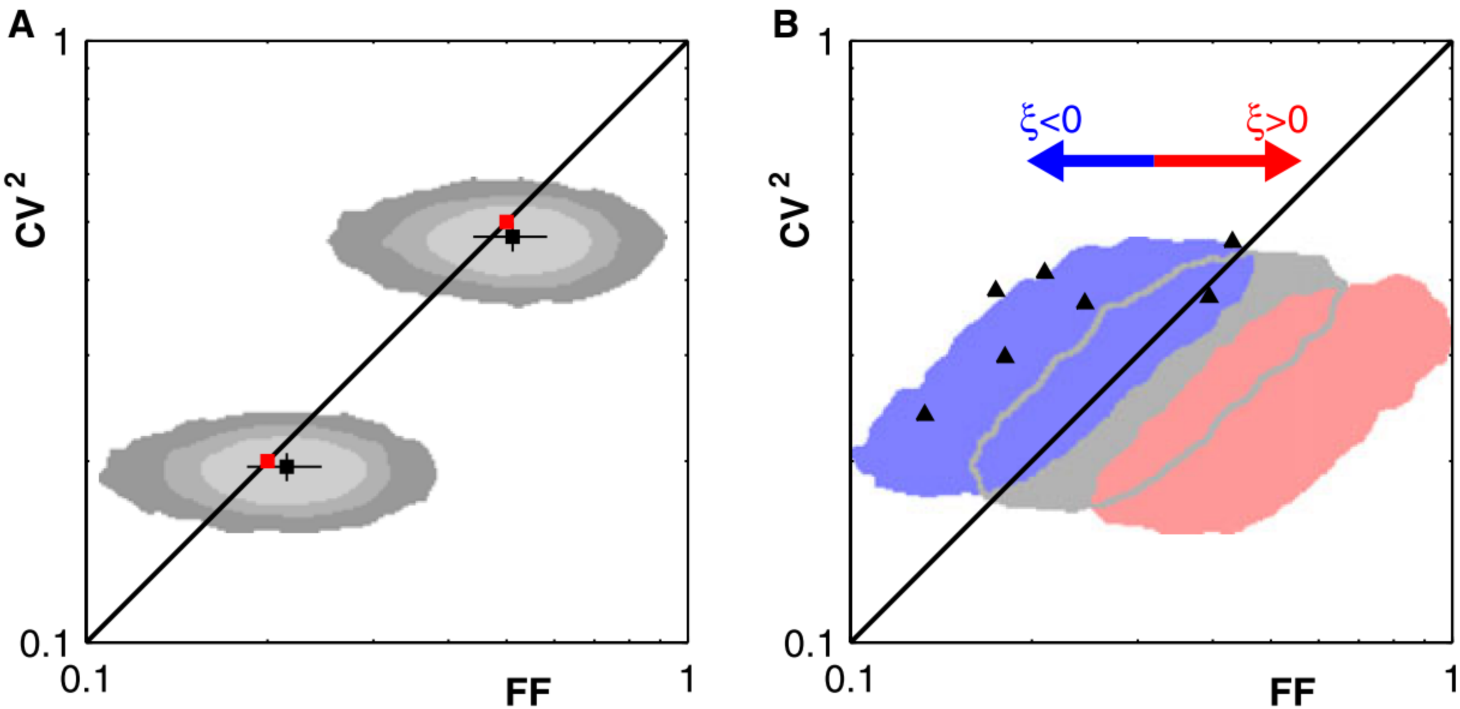
Non-renewal autoregressive point process

- *Numeric Simulation*
- log-normal
- CV = 0.5



Farkhooi, Strube-Bloss & Nawrot (2009) Phys Rev E

Non-renewal autoregressive point process



$$\lim_{T\to\infty} FF = CV^2\left(1 + 2\sum_{i=1}^{\infty} SRC_i\right)$$

Nawrot (2010) In: Grün, Rotter (eds.), Springer Series Comp Neurosci 7

Variability of neural ensemble activity

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Thought experiment:

Superimpose N realizations of a gamma renewal processes
of identical intensity and shape parameter. What happens to the CV^2,
what happens to FF of the *superimosed* process ?

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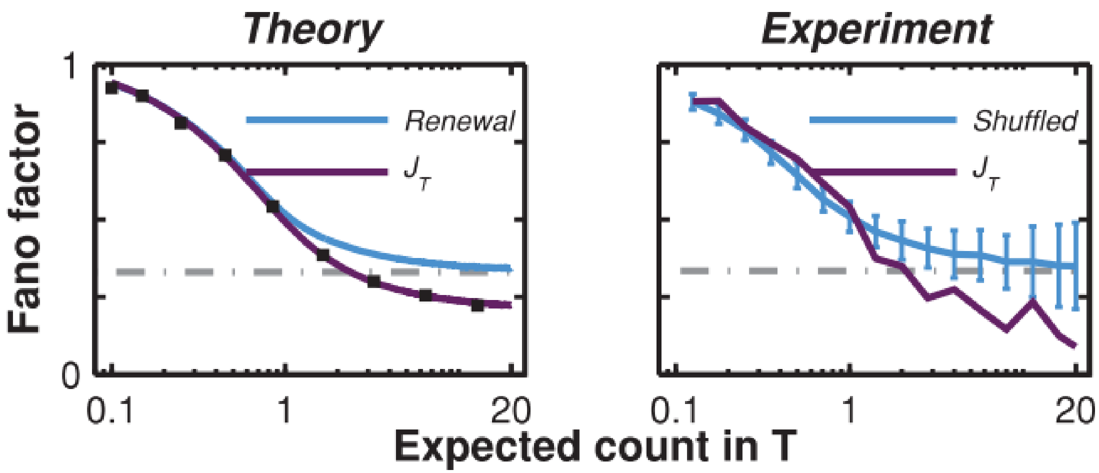
- superposition retains Fano factor, but **increases CV** and adjusts SRCs accordingly
- superposition of renewal processes results in a non-renewal process (exception: Poisson)



$$\lim_{T\to\infty} FF = CV^2\left(1 + 2\sum_{i=1}^{\infty} SRC_i\right)$$

Farkhooi, Müller & Nawrot (2011) Phys Rev E

Overview: point process model classes

constant intensity λ	*dynamic intensity λ(t)*
Poisson • exponential interval distribution • Poisson count distribution • events are uniformly distributed in time • special case of gamma process	**non-homogenous Poisson**
Renewal • iid interval distribution • $FF = CV^2$	**Rate modulated Renewal**
stationary non-Renewal • constant intensity parameter • non-trivial history dependence • serial interval correlations	

increasing importance of process history ↓

Take Home Messages

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- The Poisson model is a deficient model for neural spiking

- Short-lived negative serial interval correlations exist in stationary cortical spike trains. Neuronal spiking is generally non-renewal

- Negative serial correlations reduce variability in single neuron and population activity. This benefits reliable population coding

- Biophysical mechanisms of self-inhibition such as spike-frequency adaptation provide a likely explanation for the negative ISI correlations (see also my talk tomorrow)

References

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