Exercise 07

PLEASE HAND IN YOUR SOLUTION BEFORE **THURSDAY, 15 DEC, 8.00 A.M.**

1 Uncertainty in the angular momentum components 10 points

a. Show that

- $[L_x, L_y] = i\hbar L_z$
- $[L_y, L_z] = i\hbar L_x$
- $[L_z, L_x] = i\hbar L_y$

(See exercise 6 for the definition of the angular momentum operators L_x, L_y, and L_z.)

b. Calculate the product of the uncertainties $\Delta L_x \Delta L_y$ for the spherical harmonics Y_1^0 and $Y_1^{\pm 1}$. Are you surprised by the results? Why / why not?

c. In which special case is it possible to know exactly all three components of the angular momentum without violating the Heisenberg uncertainty principle?

d. Calculate $\langle L_x \rangle$ and $\langle L_y \rangle$ for Y_1^0.

e. Calculate $\langle L_x \rangle$ and $\langle L_y \rangle$ for $Y_1^{\pm 1}$

f. Calculate $\langle L_x \rangle^2$ and $\langle L_y \rangle^2$ for Y_1^0 and $Y_1^{\pm 1}$. (Hint: $\hat{\mathbf{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ and $\langle L_x^2 \rangle = \langle L_y^2 \rangle$)

2 Rotational dynamics 10 points

Let's consider a IBr molecule with a rotational constant B = 0,05683 cm^{-1}, where

$$B = \frac{\hbar^2}{2I}\,.$$

Let the rotational wavefunction be a superposition of the rotational eigenstates Y_1^0 and Y_2^0

$$\Psi(\varphi, \theta, t) \quad = \quad N \left[c_1 \exp\left(-i \frac{E_{l=1} t}{\hbar} \right) Y_1^0(\varphi, \psi) + c_2 \exp\left(-i \frac{E_{l=2} t}{\hbar} \right) Y_2^0(\varphi, \psi) \right] \tag{1}$$

with $c_1 = 1$ and $c_2 = 1$.

a. Normalize the wavefunction (either using Python or pen and paper).

b. Calculate the period T of the time evolution of $\Psi(\varphi, \theta, t)$.

c. Plot $|\Psi(\varphi, \theta, t)|^2$ as a function of θ at time points $t = 0$, $t = 0.25\,T$, $t = 0.50\,T$, $t = 0.75\,T$, and $t = T$ (either using Python or pen and paper).

3 Python exercise 10 points

Open a new file "HelloWorld.txt" and use a for loop to write the following lines to this file:

```
0
H
1
He
2
Hel
3
Hell
4
Hello
5
HelloW
6
HelloWo
7
HelloWor
8
HelloWorl
9
HelloWorld
10
HelloWorld!
```

3.1

Now create a list with integers from 0 to 10.

3.2

Use the following function to create a 5×11 matrix containing 5 shuffled versions of the list of integers.

```
def shuffled(arr):
        lst = list(arr)
        random.shuffle(lst)
        return lst;
```

3.3

Write this matrix to a new file called "numbers.txt":

```
7 10 0 5 8 4 6 3 2 1 9
5 3 0 6 2 8 9 10 4 7 1
1 5 7 4 3 9 8 10 0 2 6
7 6 3 8 1 10 5 9 2 0 4
6 3 2 1 5 10 9 7 0 8 4
```