Exercise 04

PLEASE HAND IN YOUR SOLUTION BEFORE **THURSDAY, 24 NOV, 8.00 A.M.**

1 Hermitian operators (15 points)

a. Consider an operator $\hat{A}$ with degenerate eigenvectors $|\phi_i\rangle$ (i.e. with eigenvectors with the same eigenvalues). Prove that any linear combination of these eigenvectors are also eigenvectors of the operator $\hat{A}$ with the same eigenvalue.

b. Prove that Hermitian operators have real eigenvalues.

c. Prove that Hermitian operators have orthogonal eigenvectors.

2 Dirac notation (10 points)

a. Consider an orthonormal basis:

$$\{|1\rangle, |2\rangle, |3\rangle\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \tag{1}$$

and the vectors:

$$|a_1\rangle = 2|1\rangle + 1|2\rangle + 1|3\rangle$$
$$|a_2\rangle = -1|1\rangle + 0|2\rangle + 1|3\rangle$$
$$|a_3\rangle = 0|1\rangle + 2|2\rangle + 3|3\rangle$$

Show that the vectors $|a_1\rangle, |a_2\rangle$ and $|a_3\rangle$ are linearly independent.
Consider the vectors:

$$|b_1\rangle = 2i|1\rangle + 0|2\rangle - 1|3\rangle$$
$$|b_2\rangle = 0|1\rangle - i|2\rangle + 3|3\rangle$$

Compute: $\langle b_1|, \langle b_2|, \langle b_1|b_2\rangle, |b_1\rangle\langle b_2|$

b. Given the operator $\hat{A}$, that acts on the vector basis as follows:

$$\hat{A}|1\rangle = 3|1\rangle + i|2\rangle$$
$$\hat{A}|2\rangle = -i|1\rangle + 1|3\rangle + 2|3\rangle$$
$$\hat{A}|3\rangle = 1|2\rangle - 2i|2\rangle$$

find the matrix representation of $\hat{A}$. Is the operator Hermitian?

3 Particle in a box (10 points)

a. Show that the wavefunctions of a particle in a box are orthonormal. Use the wavefunction:

$$\psi(x) = B \sin\left(\frac{n\pi}{L}x\right) \tag{2}$$

Use this result to normalize the wavefunction and to find the value of B.

b. Show that the average momentum of a particle in a box is zero:

$$\langle \hat{p}_x \rangle = \int_0^L \psi(x)^* \hat{p}_x \psi(x) dx = 0 \tag{3}$$

4 Python Exercise (10 points)

```
import numpy as np
import matplotlib.pyplot as plt
```

4.1

Create an array with 100 random floating values between 0 and 30.

4.2

Discretize this array using a bin width of 2. It should then contain integer values ranging between 0 and 15.

4.3

Create a histogram containing the respective number of times the value i occured in the discretized matrix. $i = 0, ..., N$. N is the highest bin.

4.4

Plot this histogram using the matplotlib function `bar()`. The x axis should not be labeled with the bins but the original values.