Exercise 01

Differential equations

PLEASE HAND IN YOUR SOLUTION BEFORE **THURSDAY, 3 NOV, 8.00 A.M.**

1.1 Classification of differential equation (10 P)

Classify the following differential equations by

- type of differetential equation (ODE or ODE)
- order of the differential equation
- linear or non-linear differential equation
- homogeneous or inhomogeneous differential equation
- constant or variable coefficients

a.

$$\frac{d^2}{dx^2}f(x) + 2\frac{d}{dx}f(x) = \sin(x)$$

b.

$$i\frac{\partial}{\partial t}\Psi = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Psi + k|\Psi|^2\Psi$$

c.

$$\frac{d^2}{dx^2}y(x) + 2xy(x) = x$$

d.

$$\frac{d^2}{dt^2}\phi(x) + \sin(\phi) = 0$$

1.2 Oscillations (5 P)

Consider the differential equation

$$\frac{d^2}{dt^2}f(t) + \omega^2 f(t) = 0 \tag{1}$$

a. Solve the equation with initial conditions

$$f(0) = 0 \tag{2}$$
$$\frac{d}{dt}f(0) = \nu_0 \tag{3}$$

with $\nu_0 \in \mathbf{R}$.

b. Solve the equation with initial conditions

$$f(0) = A \tag{4}$$
$$\frac{d}{dt}f(0) = \nu_0 \tag{5}$$

with $A, \nu_0 \in \mathbf{R}$.

c. Prove that in both cases $f(t)$ oscillates with frequency $\omega/(2\pi)$

1.3 Superposition principle (4 P)

a. Find the general solution for the differential equation

$$\frac{d^2}{dx^2}f(x) - 3\frac{d}{dx}f(x) + 2f(x) \quad = \quad 0\,. \tag{6}$$

b. Consider the linear second-order differential equation

$$\frac{d^2}{dx^2}f(x) + a_1(x)\frac{d}{dx}f(x) + a_0(x)f(x) \quad = \quad 0\,. \tag{7}$$

The equation does not have constant coefficients and there is no general, simple method for solving it. Nevertheless, because it is linear, we must have that if $f_1(x)$ and $f_2(x)$ are any two solutions, then a linear combination,

$$f(x) = c_1 f_1(x) + c_2 f_2(x)$$

where c_1 and c_2 are constant coefficients, is also a solution. Prove that $f(x)$ is a solution.