Exercise 05

Submit your exercises before next Friday at 8.00 am

1 Commutators (10 points)

1. Consider the position operator $\hat{x}$ and the momentum operator $\hat{p}$, prove that:

$$[\hat{x}, \hat{p}] = -i\hbar$$

2. The angular momentum is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

   with $\mathbf{r} = (\hat{x}, \hat{y}, \hat{z})$ and $\mathbf{p} = (\hat{p_x}, \hat{p_y}, \hat{p_z})$. Prove that:

$$\hat{L}_x = y\hat{P}_z - z\hat{P}_y$$

$$\hat{L}_y = z\hat{P}_x - x\hat{P}_z$$

$$\hat{L}_z = x\hat{P}_y - y\hat{P}_x$$

3. Prove that:

$$\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$$

$$\left[\hat{L}_y, \hat{L}_z\right] = i\hbar\hat{L}_x$$

$$\left[\hat{L}_z, \hat{L}_x\right] = i\hbar\hat{L}_y$$

2 Hermitian operators (15 points)

1. Consider an operator $\hat{A}$ with degenerate eigenvectors $|\phi_i\rangle$ (i.e. with eigenvectors with the same eigenvalues). Prove that any linear combination of these eigenvectors are also eigenvectors of the operator $\hat{A}$ with the same eigenvalue.
2. Prove that Hermitian operators have real eigenvalues.
3. Prove that Hermitian operators have orthogonal eigenvectors.