Exercise 02

Submit your files before next Friday at 8.00 am to luca.donati@fu-berlin.de

1 MonteCarlo integral (20 Points)

We learned during the tutorial how to use the MonteCarlo method to estimate the integral of a surface. We are now going to implement the method, to compute the integral of the probability distribution of the time-dependent superposition of eigenstates and check that its value, over all the space, is 1.

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \mathrm{d}x = 1$$

To implement the MonteCarlo integral, we are going to write the main function in the file userfunctions.py (under the functions eigenstates() and superposition()).

```
def montecarloIntegral(x,psi2,t,MCpoints):
#We pass to the function the x-axis, |psi(x,t)|^2, timestep
#MCpoints is the number of random points
    yMin = ... #limits of the function
    yMax = ...
    xMin = ...
    xMax = ...

    areaRect = ...

    pointsX = ... #array with MCpoints random points, from a uniform distribution,
                  #between xMin and xMax
    pointsY = ... #array with MCpoints random points, from a uniform distribution,
                  #between yMin and yMax

    phi, E = ... #eigenstates and eigenvalues using pointsX
                 #(use the function eigenstates)
    psi_r = ...  #superposition (use the function superposition)

    pointsInside = ...  #number of points "pointsY" that fall under |psi|^2

    integral = ...
    return ...
```

This function computes the integral at each time-step, then in the main program wavepacket.py, inside the for block, add the following line:

```
print montecarloIntegral(x,psi2,t,MCpoints)
```

where MCpoints is the number of points that we use to compute the MonteCarlo integral and psi2 is the square of the absolute value of the wave function (this value is returned by the function superposition).

2 Classification of differential equations (10 points)

Classify the following differential equations by:

- Type of differential equation (ODE or PDE)
- Order of the differential equation
- Linear or non-linear differential equation

- Homogeneous or inhomogeneous differential equation
- Constant or variable coefficients

$$y'' + 2y' + y = \sin(x) \tag{1}$$

$$y' + 2xy = x \tag{2}$$

$$y'' + \sin(y) = 0 \tag{3}$$

$$u_{xx}(x, y) = u_{xy}(x, y) + \exp(y) \tag{4}$$

$$u_{xx}(x, y, z) + u_{yy}(x, y, z) + u_{zz}(x, y, z) = V(x, y, z)u(x, y, z) \tag{5}$$

Notation:

$y' = \frac{dy(x)}{dx}$

$y'' = \frac{d^2y(x)}{dx^2}$

$u_{xx}(x, y) = \frac{d^2u(x,y)}{dx^2}$

$u_{xy}(x, y) = \frac{d^2u(x,y)}{dxdy}$

3 Solution of ordinary linear differential equations (10 points)

Solve the following two linear differential equations using the method of "separation of variables":

$$y' = 2x\sqrt{1 - y^2} \tag{6}$$

$$xy' = x + 2y \quad (\text{Hint} : y = ux) \tag{7}$$