Exercise 05

1 Quantum harmonic oscillator with ladder operators (10 points)

The ladder operators for the harmonic oscillator are:

$$\hat{a} = \frac{\omega m \hat{x} + i\hat{p}}{\sqrt{2\omega m\hbar}} \qquad \text{(lowering operator)}$$

$$\hat{a}^\dagger = \frac{\omega m \hat{x} - i\hat{p}}{\sqrt{2\omega m\hbar}} \qquad \text{(raising operator)}$$

The number operator is:

$$\hat{n} = \hat{a}^\dagger \hat{a}$$

1. Derive the ladder operators, starting from the Hamiltonian for the harmonic oscillator:

$$\hat{H} = \frac{m\omega^2}{2}\hat{x}^2 + \frac{\hat{p}^2}{2m}$$

   Hint: Use the relation $(u+iv)(u-iv) = u^2 + v^2 - iuv + iuv$ and find the Hamiltonian in the form $\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$.

2. Show that:

   - $\left[\hat{a}, \hat{a}^\dagger\right] = 1$
   - $[\hat{a}, \hat{a}] = \left[\hat{a}^\dagger, \hat{a}^\dagger\right] = 0$
   - $\left[\hat{n}, \hat{a}^\dagger\right] = \hat{a}^\dagger$
   - $[\hat{n}, \hat{a}] = -\hat{a}$

2 Angular momentum in quantum mechanics (10 points)

1. Show that:

$$\left[\hat{L}^2, \hat{L}_x\right] = \left[\hat{L}^2, \hat{L}_y\right] = \left[\hat{L}^2, \hat{L}_z\right] = 0$$

2. Using MATLAB, plot the first six associated Legendre polynomials $P_l^{|m|}(x)$ in the interval $[-1, 1]$. With the matlab function polar(theta,rho), plot also the representation in polar coordinates $P_l^{|m|}(\cos(\theta))$.

3 Molecular spectroscopy (10 points)

1. The infrared spectrum of $^{75}Br^{19}F$ consists of an intense line at 380 cm^{-1}. Calculate the force constant of the molecule.

2. The microwave spectrum of $H^{35}Cl$ consists of a series of equally spaced lined, separated by $6.26 \cdot 10^{11}$ H. Calculate the bond length of the molecule.

For the parts where you do not need MATLAB you can write a report by hand and deliver it at the office 35.17. For the MATLAB parts, send the .m script by email to luca.donati@fu-berlin.de before next Wednesday at noon.