Exercise 01

1 Classification of differential equations (25 points)

Classify the following differential equations by:

- Type of differential equation (ODE or PDE)
- Order of the differential equation
- Linear or non-linear differential equation
- Homogeneous or inhomogeneous differential equation
- Constant or variable coefficients

$$y'' + 2y' + y = \sin(x) \tag{1}$$

$$y' + 2xy = x \tag{2}$$

$$y'' + \sin(y) = 0 \tag{3}$$

$$u_{xx}(x,y) = u_{xy}(x,y) + \exp(y) \tag{4}$$

$$u_{xx}(x,y,z) + u_{yy}(x,y,z) + u_{zz}(x,y,z) = V(x,y,z)u(x,y,z) \tag{5}$$

Notation:

$$y' = \frac{dy(x)}{dx}$$

$$y'' = \frac{d^2y(x)}{dx^2}$$

$$u_{xx}(x,y) = \frac{d^2u(x,y)}{dx^2}$$

$$u_{xy}(x,y) = \frac{d^2u(x,y)}{dxdy}$$

2 Solution of ordinary linear differential equations (10 points)

Solve the following two linear differential equations using the method of "separation of variables":

$$y' = 2x\sqrt{1-y^2} \tag{6}$$

$$xy' = x + 2y \quad (\text{Hint} : y = ux) \tag{7}$$

3 Self-adjoint operators and commutators (25 points)

Prove that:

1. If $\hat{A}$ and $\hat{B}$ are self-adjoint, then $\hat{C} = \hat{A} + \hat{B}$ is self-adjoint.
2. If $\hat{A}$ and $\hat{B}$ are self-adjoint, then $\hat{C} = \hat{A}\hat{B}$ is only self-adjoint, if $\hat{A}$ and $\hat{B}$ commute $\left(\left[\hat{A},\hat{B}\right] = 0\right)$.
3. If $\hat{A}$ is self-adjoint, then $\hat{A}^n$ is self-adjoint.
4. If $\hat{A}$ is self-adjoint, then $\exp(\hat{A})$ is self-adjoint.
5. If $f(x)$ is an analytic function, $\hat{A}$ and $\hat{B}$ are self-adjoint, then $[\hat{A}, f(\hat{B})] = f'(\hat{B})[\hat{A},\hat{B}]$

NB: A function $f(x)$ is said to be analytic around a point x_0 if $f(x)$ can be expanded in a power series around x_0. If $f(x)$ is analytic, then its Taylor series converges to $f(x)$ for x in a neighborhood of x_0.

You can write a report by hand and deliver it at the office 35.17 or write an electronic document and send it to luca.donati@fu-berlin.it before next Wednesday at noon.