Exercise 05

Name:	Matriculation number:
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Results:

5.1	of 10 P
5.2	of 4 P
5.3	of 2 P
5.4	of 10 P
total	of 26 P

General instructions

- Submit your solution to Oliver Lemke (R. 35.17) before **Thursday 02. June at 8.15 am**.
- Fill out this cover sheet and submit it along with your solution.
- Show how you arrived at your answer.

5.1 Bose-Einstein and Fermi-Dirac statistics (10 P)

If we assume a small interval of energy states $[\epsilon_i, \epsilon_i + d\epsilon]$ with g_i nearly degenerated states and N_i particles populating these states, the population of the energy level can be calculated using the Bose-Einstein statistics (bosons) or the Fermi-Dirac statistic (fermions) respectively.

(a) For fermions the statistic can be calculated easily as the maximal occupation number of each state is 1. Calculate the number of possible configurations for $N_i = 6$ and $g_i = 8$ if the particles are distinguishable and indistinguishable respectively.

(b) For bosons the occupation number is not limited. Repeat (a) for indistinguishable bosons

(c) The distribution functions for bosons and fermions, which define the average occupation number for each state, are defined as:

$$f(BE) = \frac{N_i}{g_i} = \frac{1}{\mathrm{e}^{\beta(\epsilon_i - \mu)} - 1} \tag{1}$$

$$f(FD) = \frac{N_i}{g_i} = \frac{1}{\mathrm{e}^{\beta(\epsilon_i - \mu)} + 1} \tag{2}$$

where μ is the chemical potential which is for bosons always smaller than ϵ_i. Plot $f(BE)$ and $f(FD)$ for different values of $\epsilon_i - \mu$ and explain the outcome.

(d) Calculate the border cases $\epsilon_i >> \mu$; $\epsilon_i = \mu$ and $\epsilon_i << \mu$ for both statistics

5.2 Energy fluctuation in the canonical ensemble (4 P)

The energy fluctuation in a canonical ensemble is defined as

$$\Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2 \tag{3}$$

where $\langle E \rangle$ is defined as

$$\langle E \rangle = \frac{1}{Q} \sum_j E_j \mathrm{e}^{-\beta E_j}. \tag{4}$$

Show that for this case

$$\Delta E^2 = k_b T^2 c_V$$

is fulfilled.

5.3 Temperature dependent energy difference (2 P)

Consider an energy scheme with two energy levels. The distance between this levels is k_bT. How big is the population ratio for $T = 0$ K, 298 K and 1000 K?

5.4 Gas in a macroscopic box (10 P)

For a gas in a cubic box the energy levels are defined as

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2). \tag{5}$$

If we consider a macroscopic box, the energy levels are very close to each other. Therefore, all combinations of n_x, n_y and n_z with an energy E can be found on a sphere with a radius of R

(a) Show that

$$R = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{2L}{h}\sqrt{2mE}$$

(b) The number of states that are located in the interval between 0 and E (state distribution) can be described by the volume of $\frac{1}{8}$ of a sphere. Why?

(c) Write down the formula for the state distribution

(d) Show that the number of states per molecule for an ideal gas is given by

$$\frac{W}{T} = \frac{4\pi}{3}\frac{1}{p}(k_bT)^{\frac{5}{2}}(2m)^{\frac{3}{2}}$$

(e) Calculate the number of states per molecule for hydrogen at room temperature and normal pressure.

(f) Repeat the calculation for helium. Compare your results with (e). What do you observe? Explain.