Project 01

Langevin dynamics and Markov state models

Keywords: MD integrators, Markov state models
Deadline: Please, hand in your report by **Tuesday, 29 July 2014**

1 Introduction

A variant of molecular dynamics is stochastic dynamics. In this method the fast degrees of freedom of the system (usually the solvent molecules) are not modeled explicitly, but their effect on the slow degrees of freedom (usually the solute) is take into account by introducing a heat bath into the equations of motion:

$$m\ddot{\mathbf{x}}(t) \quad = \quad -\nabla V(\mathbf{x}) - \gamma\, m\, \dot{\mathbf{x}}(t) + \sqrt{2\gamma\frac{m}{\beta}}\, R(t)\,. \tag{1}$$

where m is the mass and $V(x)$ is the potential energy functions. $\gamma\, m\, \dot{\mathbf{x}}(t)$ is a friction term which decreases the velocity of the particle. It is counterbalanced a random noise term $\sqrt{2\gamma\frac{m}{\beta}}\, R(t)$ which models random kick from the solvent. Note the friction term and the related via γ, which has units of $1/t$. $R(t)$ generates random forces from a normal distribution with mean

$$\langle R(t)\rangle \quad = \quad 0 \tag{2}$$

and variance

$$\langle R(t), R(t')\rangle \quad = \quad \delta(t-t')\,. \tag{3}$$

Eq. 1 is called the Langevin equation. The dynamics of this equation can be simulated using the generalized Verlet algorithm (Ref. 2).

2 Outline of the project

1. Implement the generalized Verlet algorithm for Langevin Dynamics. (Use the program from exercise 02 as a starting point).
2. Test the program for a harmonic potential energy function

$$V(x) \quad = \quad \frac{1}{2}k(x-x_0)^2 \tag{4}$$

   - Plot the total energy, the kinetic energy, and the potential energy as a function of time.
   - Verify that $R(t)$ satisfies eqs. 2 and 3.
3. Vary γ and plot the phase-space distribution (i.e. $p(x,v)dvdx$, where x is the position and v is the velocity, see Fig. 14.6 in Schlick)
4. Use your program to sample a suitable more one-dimensional potential energy function.
5. Construct a Markov state model by discretizing the x-coordinate into microstates. Account for detailed balance by counting each transition $i \to j$ also as a transition $j \to i$.
   - Plot the first four left and right eigenvectors.
   - Compare the first left eigenvector to the Boltzmann distribution.
   - Plot the eigenvalue spectrum.
   - Determine the implied time scales of the slow processes.
6. Vary the friction coefficient γ in your simulation. What effect does this have on the eigenvectors and the implied timescales of the Markov state model?

3 Literature

1. Lecture notes and exercises.
2. Chapter 14 "Molecular dynamics: further topics" in T. Schlick, "Molecular Modeling and Simulation - an Interdisciplinary Guide", 2nd edition, Springer 2010.
3. Chapters 4 in: D. Frenkel, B. Smit, "Understanding Molecular Simulation - From Algorithms to Applications", Academic Press, Elsevier (USA) 2002.
4. Chapter 2 and 3 in: G. R. Bowman, V. S. Pande, F. Noé "An Introduction to Markov State Models and Their Application to Long Timescale Molecular Simulation", Springer, Dordrecht (2014)
5. J.-H. Prinz et. al. "Markov models of molecular kinetics: Generation and validation", *J. Chem. Phys.*, **134**, p. 174105 (2011)
6. M. Senne et. al. "EMMA: A Software Package for Markov Model Building and Analysis", *J. Chem. Theory Comput.*, **8**, p. 2223-2238 (2012)

Project requirements

- Describe the results of the your project in a report of 5 to 12 pages (font size 11 pt).
- The report should also contain a short description of the theory and the methods. If you implement an algorithm, briefly describe the algorithm.
- Hand in your program along with the report.
- You will present the results of your project in a 20-minute presentation, after which we will have about 10 minutes time for discussion.
- Besides questions which are directly related to the results of the project, the discussion will also cover the theory of the project and the course content which is relevant to the project.
- If you run into problems during the project or have questions, please contact us
  - Bettina Keller, bettina.keller@fu-berlin.de
  - Francesca Vitalini, francesca.vitalini11@gmail.com
- During the lecture hours (Thu, 2.15 - 3.45 pm) one of us will be in the library to answer questions.