Exercise 01

Numerical Approximations

1.1 Series limit (10 Points)

In this exercise, you can practice the use of for-loops and do-while-loops. Consider the convergent series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1}$$

(a) Write a program that calculates the series using a for-loop and then vectorize the script. Pick a real number x and test the convergence of the series. (Hint: to implement the series, you have to replace infinity by a number N).

(b) Write a program which numerically estimates the limit of the series by evaluating the first N terms. Plot the estimate as a function of N up to $N = 20$.

(c) Write a program which evaluates the series until the increment is less than a pre-defined threshold. How many terms are needed until the increment is less than 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}?

(d) Often one does not know in advance how long it will take until a certain threshold is met. To prevent a program from running too long, one stops the calculation if either the threshold or a maximum number of iterations is reached. Modify the program from (c) such that it stops if one of the following conditions is met:

- the increment is less than a predefined threshold,
- a maximum number of terms have been evaluated.

1.2 Monte Carlo 1 (8 Points)

(a) Generate two sequences of 1000 random real numbers drawn from the interval $[-5, 5]$.

(b) Interpret the two sequences as x and y coordinates. Produce a scatter plot.

(c) Add a circle with radius $r = 5$ and center $(0, 0)$ to the scatter plot

(d) Numerically estimate the area of the circle as

$$A_{circle} \approx \frac{N}{N_{tot}} \cdot A_{square}$$

where N is the number of data points within the square and N_{tot} is the total number of random data points.

(e) Increase the number of random data points. Does the result improve?

1.3 Monte Carlo 2 (8 Points)

Consider the function

$$f(x) = (x-3)^3 + 2x$$

(a) Evaluate the integral $\int_2^6 f(x)\mathrm{d}x$ using a Monte Carlo method. Consider a rectangle defined by the points $\{[2,0], [6,0], [6, f(6)], [2, f(6)]\}$, then generate $N_{tot} = 100$ random points within the rectangle and count how many points N fall under the curve.
(Hint: draw N_{tot} random numbers from the interval $[2, 6]$ for the x-values, and N_{tot} random

numbers from the interval $[0, f(6)]$ for the y-values).
Finally, the value of the integral is

$$\int_2^6 f(x)\mathrm{d}x \approx \frac{N}{N_{tot}} \cdot A_{rectangle}$$

(b) Increase the number of points N_{tot} and plot the integral result as a function of N_{tot}. Compare the estimate with the exact value of the integral.

(c) Consider now the interval $[0, 6]$ and edit the program properly to evaluate the integral

$$\int_0^6 f(x)\mathrm{d}x$$

1.4 Neighbor lists (5 Points)

(a) Generate two sequences of $N = 1000$ random real numbers drawn from the interval $[-5, 5]$.

(b) For each datapoint, write out the list of neighbors within a radius $r = 0.5$.

(c) Write a matrix M of size $N \times N$ where each element M_{ij} is 0 if the distance between the point i and j is larger than r, otherwise is 1.

(d) What is the average number of neighbors for each datapoint?

For each exercise write a script and save it as ".m" file, then send them to luca.donati@fu-berlin.de
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